

Lecture 8

ACCELERATOR PHYSICS

Melbourne

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Lecture8 – Accelerating Cavities I- contents

- ◆ Necessary conditions for acceleration
- ◆ Waves in free space
- ◆ Two travelling waves in a guide.
- ◆ A transverse electric (H) mode
- ◆ Phase velocity and Group velocity
- ◆ Transverse magnetic modes
- ◆ Transit time factor
- ◆ The cylindrical cavity

Recap of previous lecture

Longitudinal dynamics II

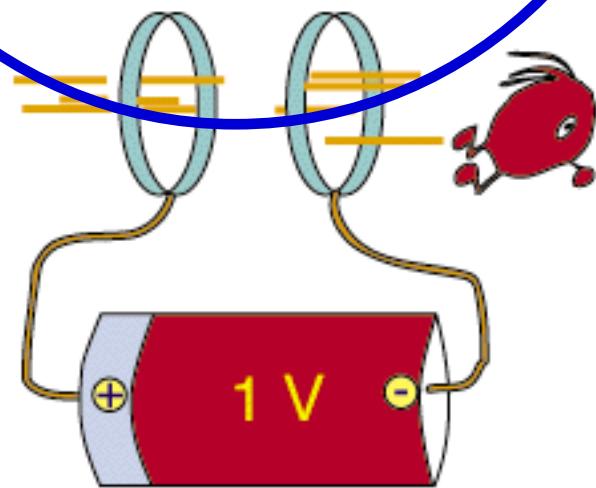
- ◆ Transition - does an accelerated particle catch up - it has further to go
- ◆ Phase jump at transition
- ◆ Synchrotron motion
- ◆ Synchrotron motion (continued)
- ◆ Large amplitudes
- ◆ Buckets
- ◆ Adiabatic capture
- ◆ A chain of buckets

Maxwell forbids this!

$$\nabla \times \mathbf{E} = -\frac{dB}{dt}$$

◆ Become in its integral form

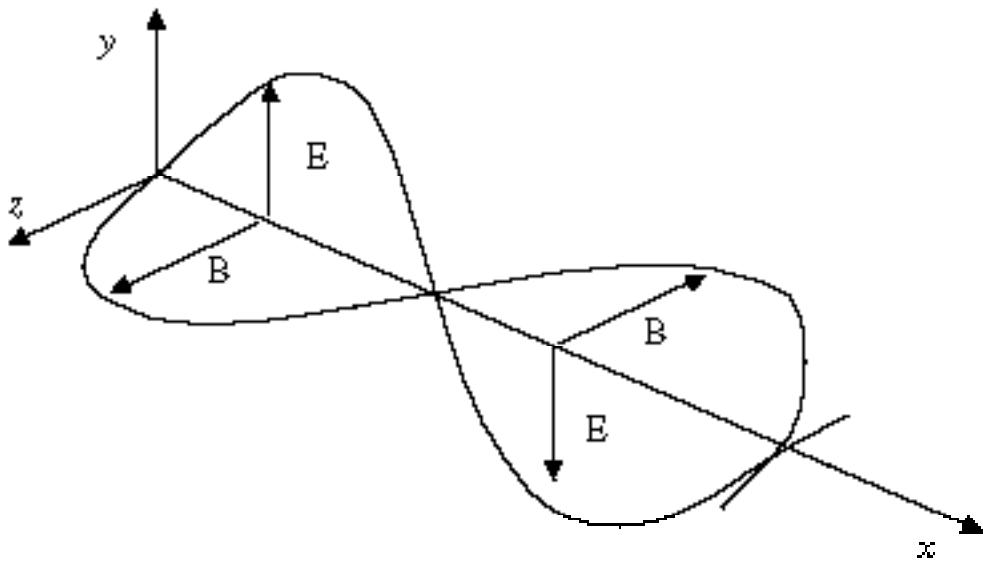
$$\int_{\Gamma} \mathbf{E} \cdot d\mathbf{s} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$$



- ◆ Hence there can be no acceleration without time dependent magnetic field
- ◆ We also see how time dependent flux may accelerate

◆ BattAcc.gif

Waves in free space



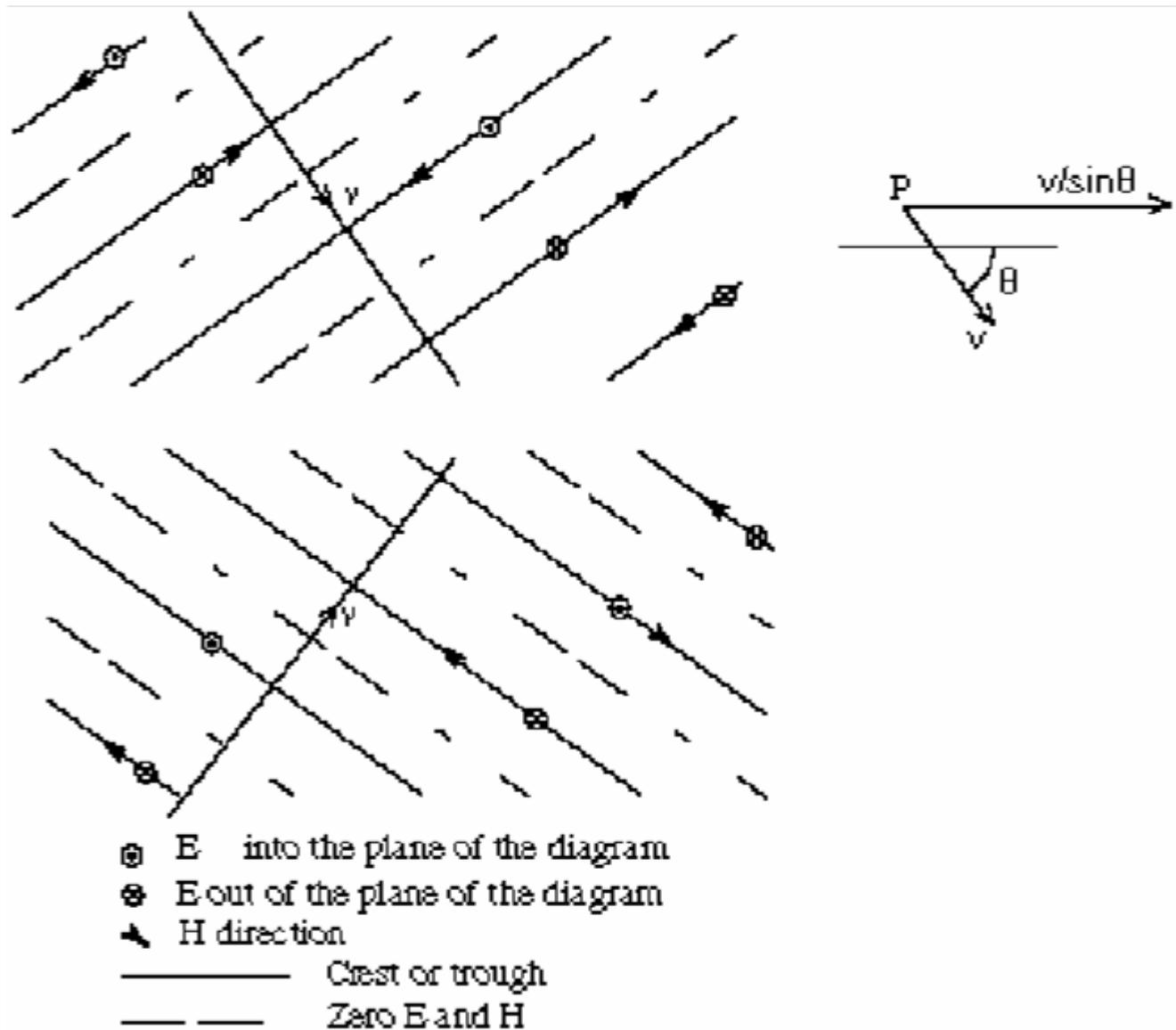
$$v = c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$v = \frac{1}{\sqrt{\epsilon_0 K_e \mu_0 K_m}}$$

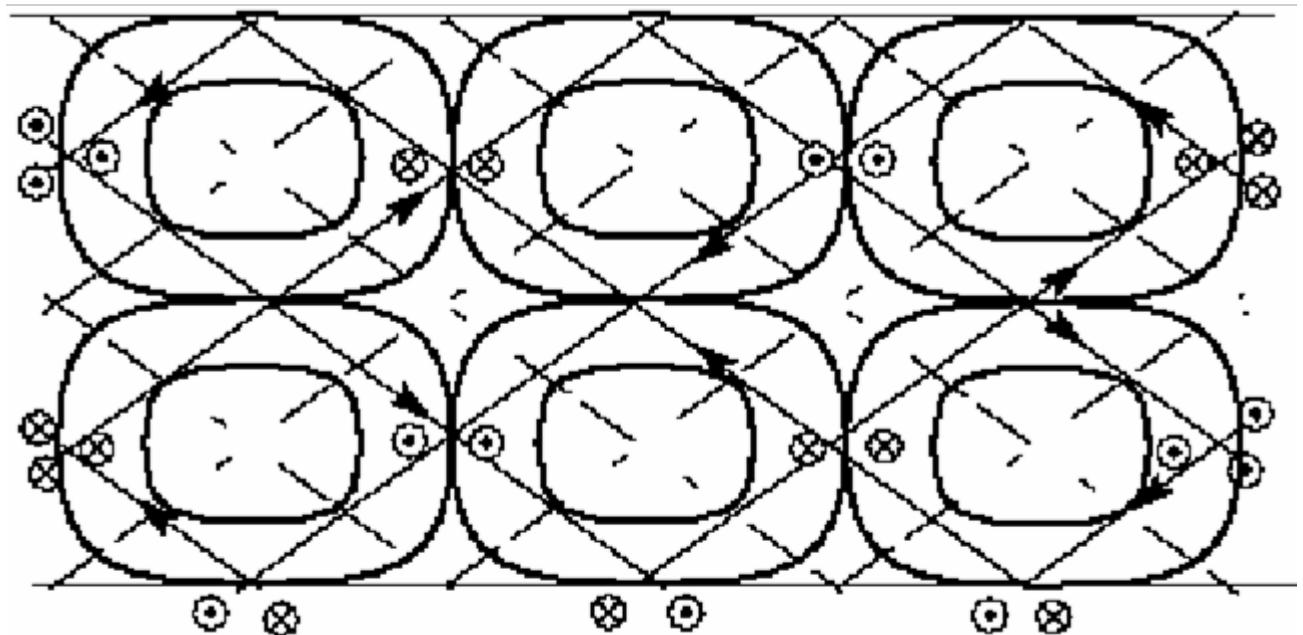
$$\frac{E}{H} = 376.6 \sqrt{\frac{K_m}{K_e}} \text{ ohms}$$

$$\mathbf{P} = (\mathbf{E} \times \mathbf{H}) \text{ watts.m}^{-2}$$

Two travelling waves in a guide.



A transverse electric (H) mode (formed by superposition of the two waves)



$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}$$

Group velocity

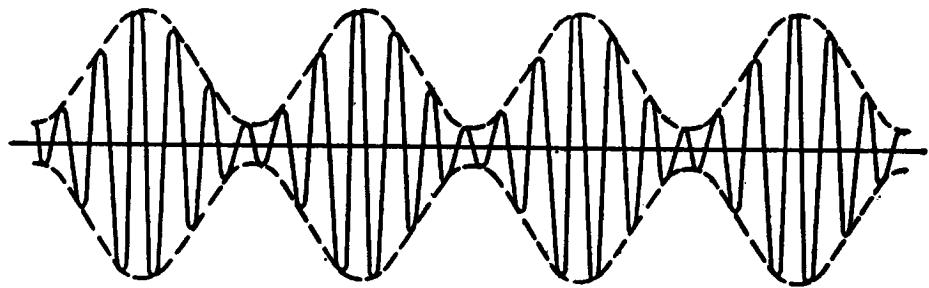


FIG. 59.—Beats.

$$\begin{aligned}
 E &= E_0 \sin[(k + dk)x - (\omega + d\omega)t] \\
 &\quad + E_0 \sin[(k - dk)x - (\omega - d\omega)t] \\
 &= 2E_0 \cos[kx - \omega t] \sin[dkx - d\omega t]
 \end{aligned}$$

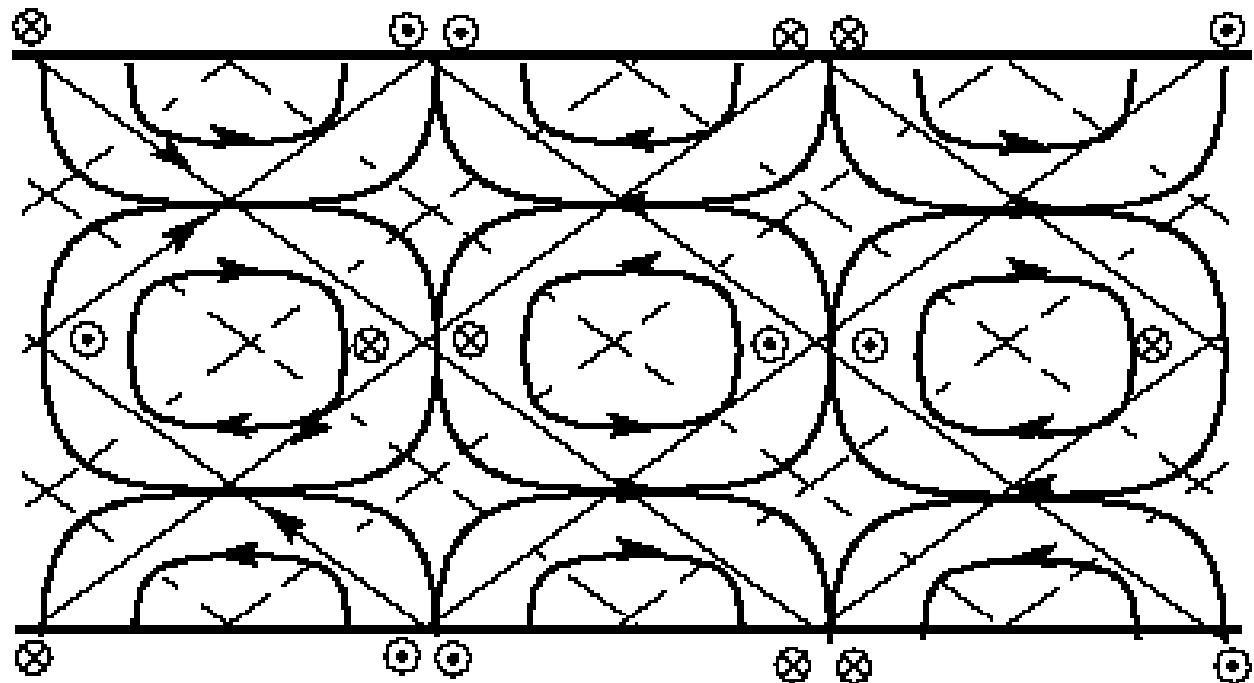
$$\begin{aligned}
 f_1(x, t) &= \sin[kx - \omega t] & k = 2\pi/\lambda \\
 kx - \omega t &= \text{const}
 \end{aligned}$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x}$$

$$v_p = -\frac{\partial f_1(x, t)/\partial t}{\partial f_1(x, t)/\partial x} = \frac{\omega}{k}$$

$$\begin{aligned}
 f_2(x, t) &= \sin[dkx - d\omega t] & v_g = -\frac{\partial f_2(x, t)/\partial t}{\partial f_2(x, t)/\partial x} = \frac{d\omega}{dk} \\
 x\delta k - t\delta\omega
 \end{aligned}$$

Transverse magnetic (E) \parallel modes



⊗ H into the plane of the diagram

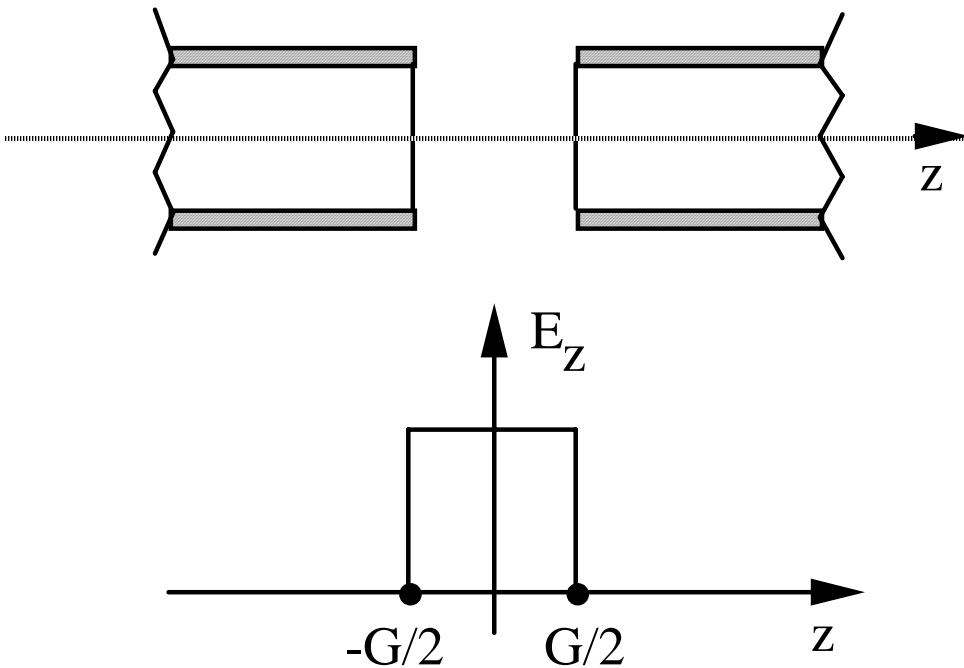
⊗ H out of the plane of the diagram

▲ E direction

— Crest or trough

— — Zero E and H

Transit time factor



$$V = \int_{-G/2}^{+G/2} E_o \cos(\omega t + \phi) dz = E_0 G \left(\frac{\sin \frac{\omega G}{2\beta c}}{\frac{\omega}{2\beta c}} \right) \cos \phi = E_0 G(\Gamma) \cos \phi$$

$$V = \int_{-G/2}^{+G/2} E_o \cos(\omega t + \phi) dz = E_0 G \left(\frac{\sin \frac{\omega G}{2\beta c}}{\frac{\omega}{2\beta c}} \right) \cos \phi = E_0 G(\Gamma) \cos \phi$$

$$\Gamma = \frac{\sin \theta / 2}{\theta / 2} .$$

Cavity resonators

$$\begin{cases} \nabla \cdot E = 0 ; \quad \nabla \cdot H = 0 \\ \nabla \times E = -\mu \frac{\partial H}{\partial t} ; \quad \nabla \times H = \sigma E + \epsilon \frac{\partial E}{\partial t} \end{cases}$$

$$\nabla^2 E = \mu \sigma \frac{\partial E}{\partial t} + \epsilon \mu \frac{\partial^2 E}{\partial t^2}$$

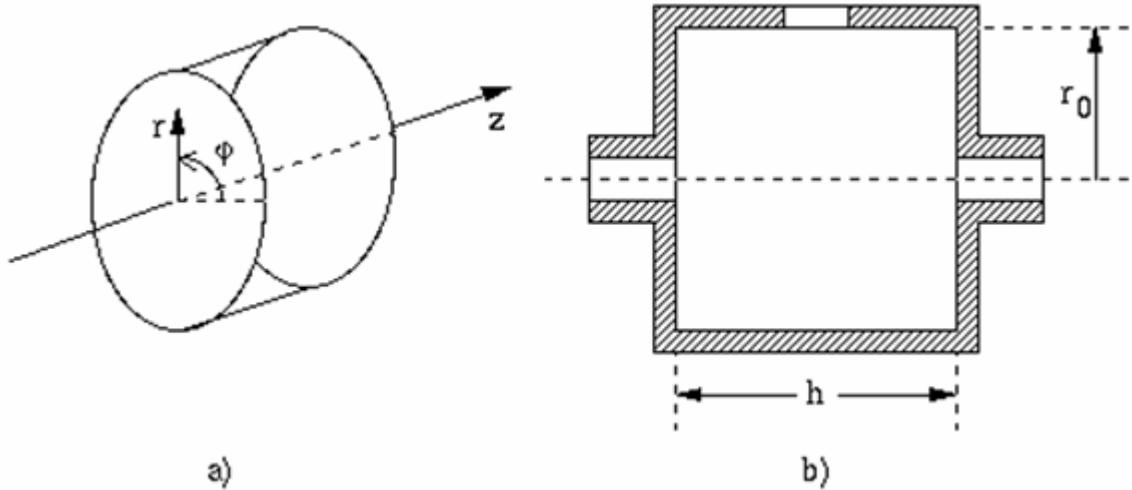
- ◆ • **$n \times E = 0$ – because the E field should be normal to the perfectly conducting walls.**
- ◆ **Assume we can separate out a time dependent solutions**

$$a_M = e^{-\frac{\omega_M}{2Q}t} \{A_1 \cos \Omega_M t + A_2 \sin \Omega_M t\}$$

- ◆ **leaving a spatial solution:**

$$\begin{cases} \nabla^2 E + \Lambda^2 E = 0 \\ \nabla \cdot E = 0 \\ \eta \times E = 0 \text{ on the boundary} \end{cases}$$

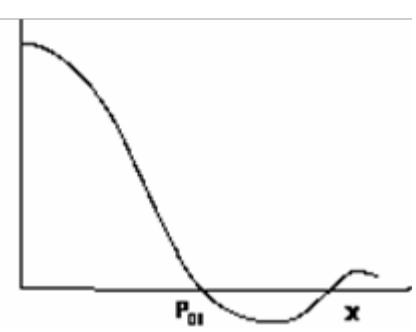
The cylindrical cavity



$$\nabla^2 \xi = \frac{\partial^2 \xi}{\partial r^2} + \frac{1}{r} \frac{\partial \xi}{\partial r} + \frac{\partial^2 \xi}{\partial z^2}$$

◆ $\mathbf{Ez} = \mathbf{F(r)} \cdot \mathbf{j(z)} ; \mathbf{Er} = \mathbf{Y(r)} \cdot \mathbf{f(z)}$

$$\begin{cases} E_z = E_0 J_0 \left(\frac{P_{0\ell}}{r_0} r \right) \cos \frac{m\pi}{h} z \\ E_r = E_0 \frac{m\pi}{P_{0\ell}} \frac{r_0}{h} J_1 \left(\frac{P_{0\ell}}{r_0} r \right) \sin \frac{m\pi}{h} z \\ \Lambda_{0\ell m}^2 = \left(\frac{P_{0\ell}}{r_0} \right)^2 + \left(\frac{m\pi}{h} \right)^2 \end{cases}$$



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