Electron beam dynamics in storage rings

Lecture 10 - Melbourne

Synchrotron radiation
and its effect on electron dynamics

Lecture 10: Synchrotron radiation

Lecture 11: Radiation damping

Lecture 12: Radiation excitation

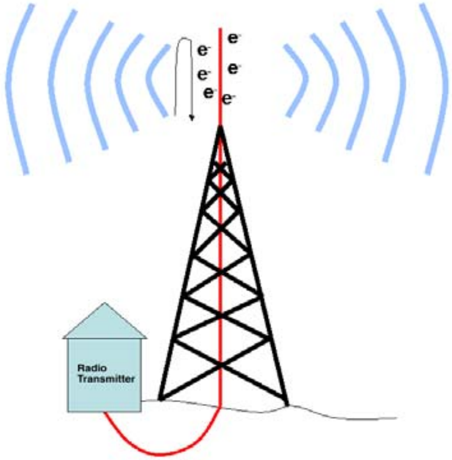
Summary of last lecture – Cavities II

- Conducting surfaces
- Quality Factor, Filling Time and Shunt Impedance
- Corrugated structures
- Dispersion in a waveguide
- Iris loaded wavequide
- Standing-wave and travelling wave structures.
- Feeding, coupling and tuning structures.
- Different modes

Contents

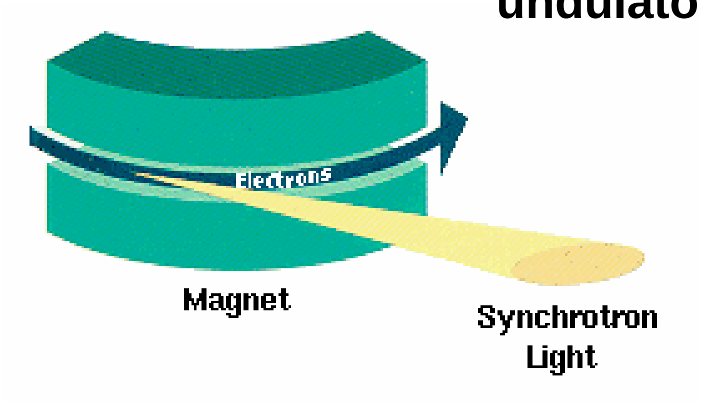
What is synchrotron radiation

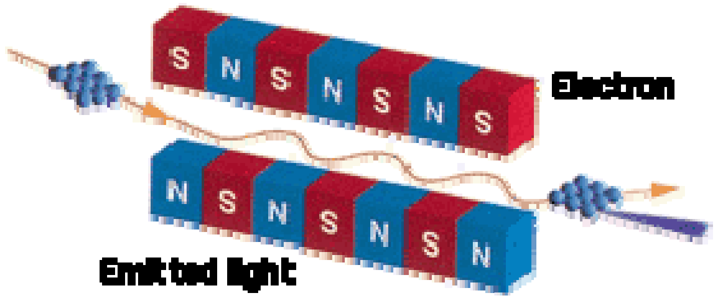
Electromagnetic radiation is emitted by charged particles when accelerated



The electromagnetic radiation emitted when the charged particles are accelerated radially (v ⊥ a) is called **synchrotron radiation**

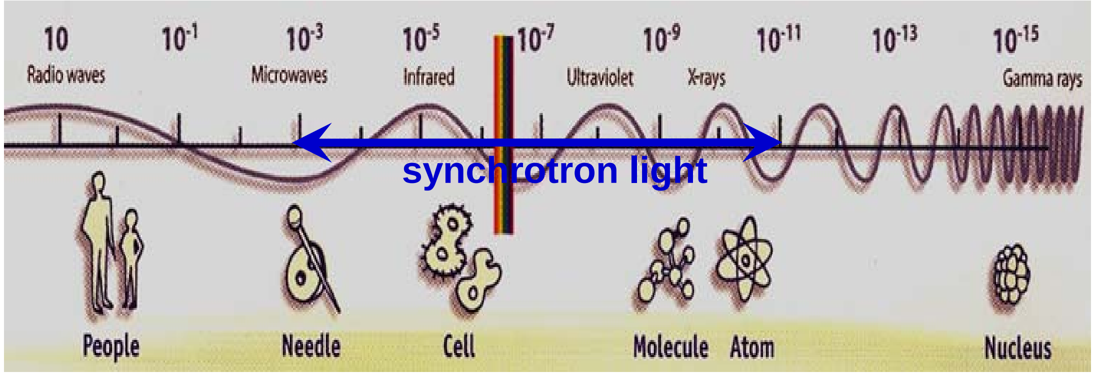
It is produced in the synchrotron radiation sources using bending magnets undulators and wigglers





Why are synchrotron radiation sources important

Broad Spectrum which covers from microwaves to hard X-rays: the user can select the wavelength required for experiment



High Flux and High Brightness: highly collimated photon beam generated by a small divergence and small size source (partial coherence)

High Stability: submicron source stability

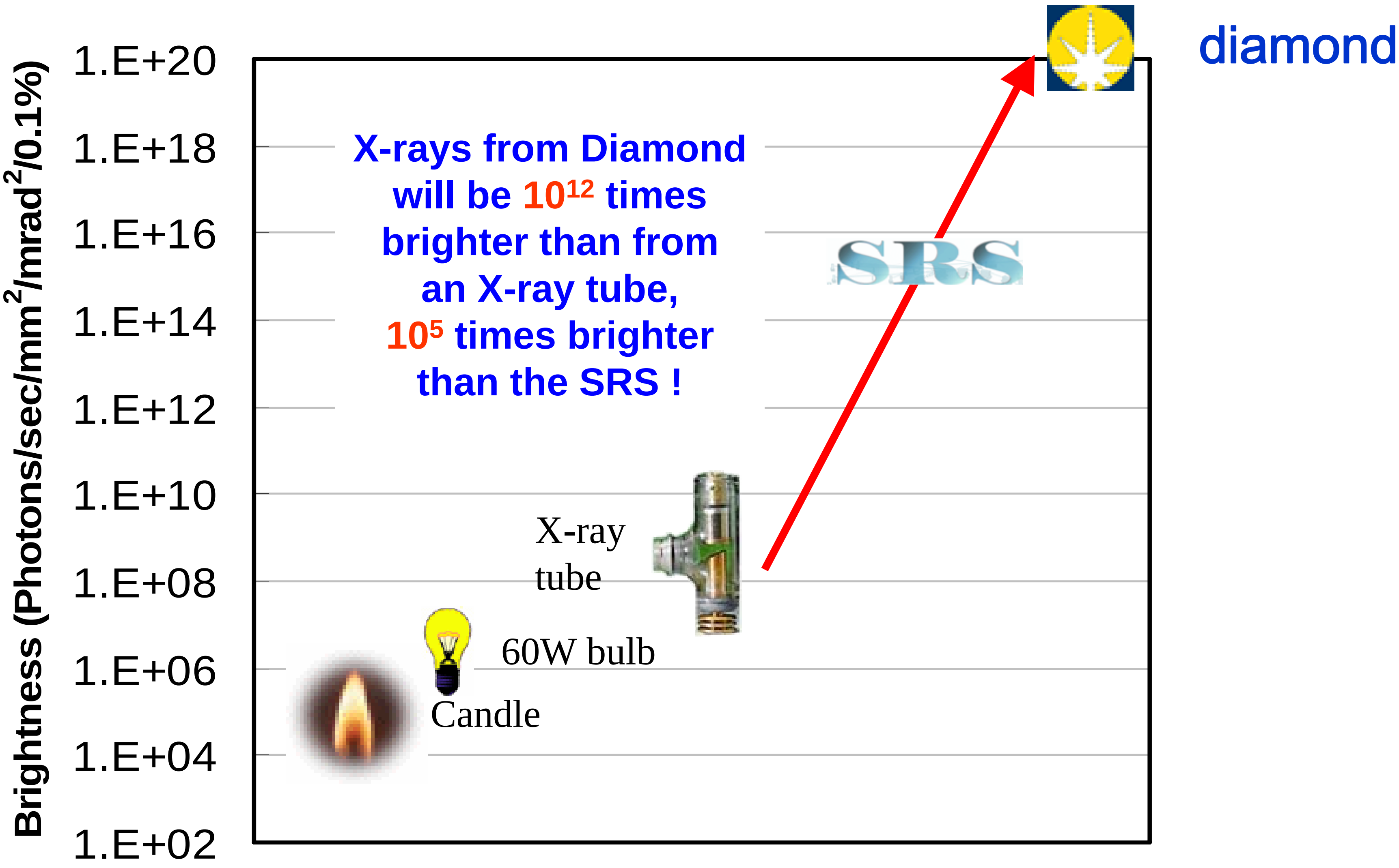
Polarisation: both linear and circular (with IDs)

Pulsed Time Structure: pulsed length down to tens of picoseconds allows the resolution of processes on the same time scale

Flux = Photons / (s • BW)

Brightness = Photons / (s • mm² • mrad² • BW)

Brightness

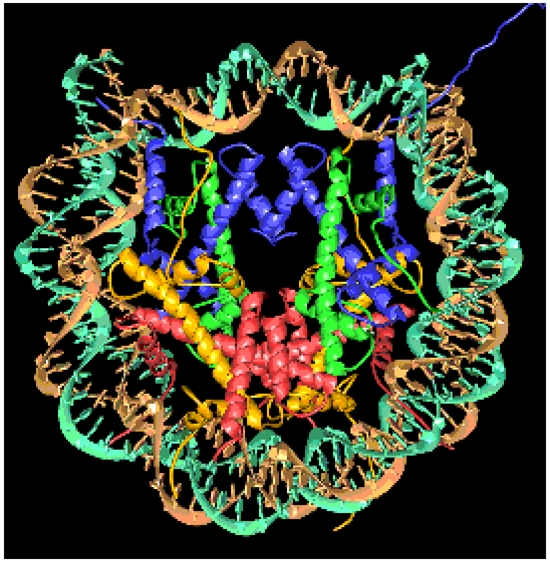

diamond
Brightness (Photons/sec/mm²/mrad²/0.1%)
1.E+20
1.E+18
1.E+16
1.E+14
1.E+12
1.E+10
1.E+08
1.E+06
1.E+04
1.E+02
X-rays from Diamond will be 10¹² times brighter than from an X-ray tube, 10⁵ times brighter than the SRS !
SRS
X-ray tube
60W bulb
Candle

Applications

Medicine, Biology, Chemistry, Material Science, Environmental Science and more

Biology

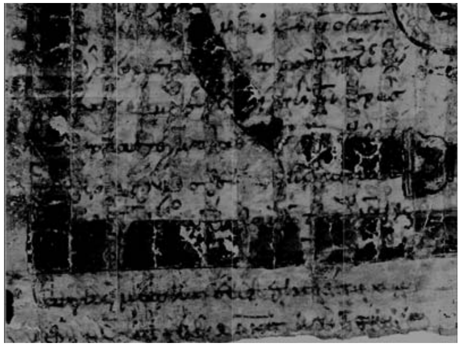
Reconstruction of the 3D structure of a nucleosome with a resolution of 0.2 nm



The collection of precise information on the molecular structure of chromosomes and their components can improve the knowledge of how the genetic code of DNA is maintained and reproduced

Archeology

A synchrotron X-ray beam at the SSRL facility illuminated an obscured work erased, written over and even painted over of the ancient mathematical genius Archimedes, born 287 B.C. in Sicily.

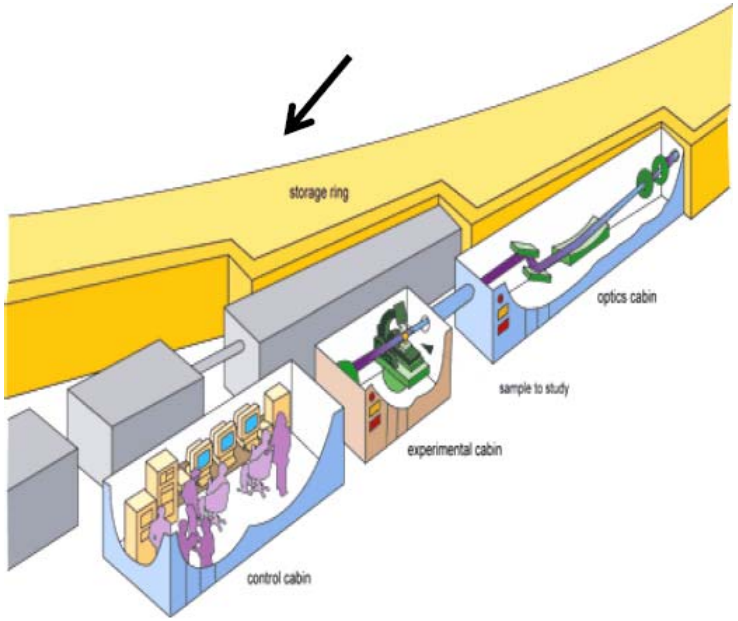


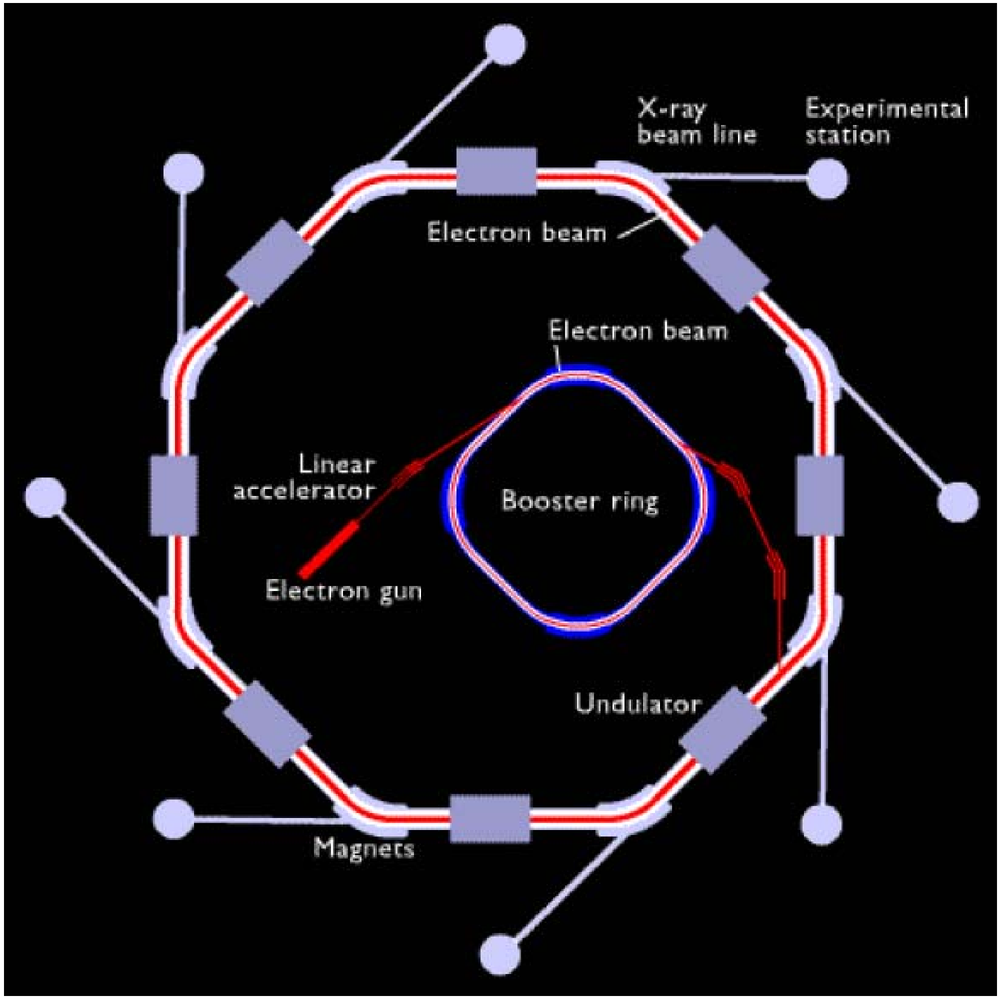
X-ray fluorescence imaging revealed the hidden text by revealing the iron contained in the ink used by a 10th century scribe. This x-ray image shows the lower left corner of the page.

Layout of a synchrotron radiation source

Electrons are generated and accelerated in a <u>linac</u>, further accelerated to the required energy in a <u>booster</u> and injected and stored in the <u>storage ring</u>

The circulating electrons emit an intense beam of synchrotron radiation which is sent down the beamline





Synchrotron Light Sources

Lienard-Wiechert Potentials (I)

The equations for vector potential and scalar potential

$$\overline{\nabla}^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\frac{\rho}{\varepsilon_0} \qquad \overline{\nabla}^2\overline{A} - \frac{1}{c^2}\frac{\partial^2\overline{A}}{\partial t^2} = -\frac{\overline{J}}{c^2\varepsilon_0}$$

with the current and charge densities of a single charged particle, i.e.

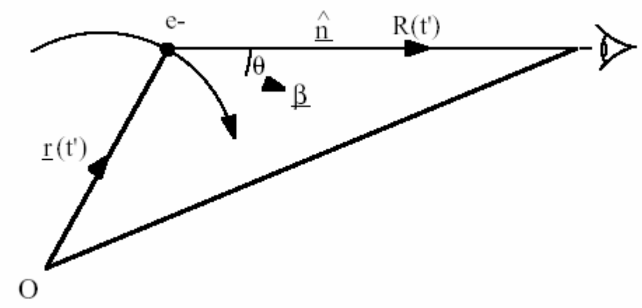
$$\rho(\overline{x},t) = e\delta^{(3)}(\overline{x}-\overline{r}(t)) \qquad \overline{J}(\overline{x},t) = e\overline{v}(t)\delta^{(3)}(\overline{x}-\overline{r}(t))$$

have as solution the Lienard-Wiechert potentials

$$\Phi(\overline{x},t) = \frac{1}{4\pi\varepsilon_0}\left[\frac{e}{(1-\overline{\beta}\cdot\overline{n})R}\right]_{ret} \qquad \overline{A}(\overline{x},t) = \frac{1}{4\pi\varepsilon_0 c}\left[\frac{e\overline{\beta}}{(1-\overline{\beta}\cdot\overline{n})R}\right]_{ret}$$

$[\ ]_{ret}$ means computed at time t'

$$t = t' + \frac{R(t')}{c}$$



Lineard-Wiechert Potentials (II)

The electric and magnetic fields generated by the moving charge are computed from the potentials

$$\overline{E} = -\nabla V - \frac{\partial \overline{A}}{\partial t} \qquad \overline{B} = \nabla \times \overline{A}$$

and are called Lineard-Wiechert fields

$$\overline{E}(\overline{x},t) = \frac{e}{4\pi\varepsilon_0}\left[\frac{\overline{n}-\overline{\beta}}{\gamma^2(1-\overline{\beta}\cdot\overline{n})^3 R^2}\right]_{ret} + \frac{e}{4\pi\varepsilon_0 c}\left[\frac{\overline{n}\times(\overline{n}-\overline{\beta})\times\dot{\overline{\beta}}}{(1-\overline{\beta}\cdot\overline{n})^3 R}\right]_{ret} \qquad \overline{B}(\overline{x},t) = \frac{1}{c}\left[\overline{n}\times\overline{E}\right]_{rit}$$

velocity field acceleration field

$$\propto \frac{1}{R} \qquad \vec{E} \perp \overline{B} \perp \hat{n}$$

Power radiated by a particle on a surface is the flux of the Poynting vector

$$\overline{S} = \frac{1}{\mu_0}\overline{E}\times\overline{B} \qquad \Phi_\Sigma(\overline{S})(t) = \iint_\Sigma \overline{S}(\overline{x},t)\cdot\overline{n}\, d\Sigma$$

Angular distribution of radiated power

$$\frac{d^2P}{d\Omega} = (\overline{S}\cdot n)(1-\overline{n}\cdot\overline{\beta})R^2$$

radiation emitted by the particle

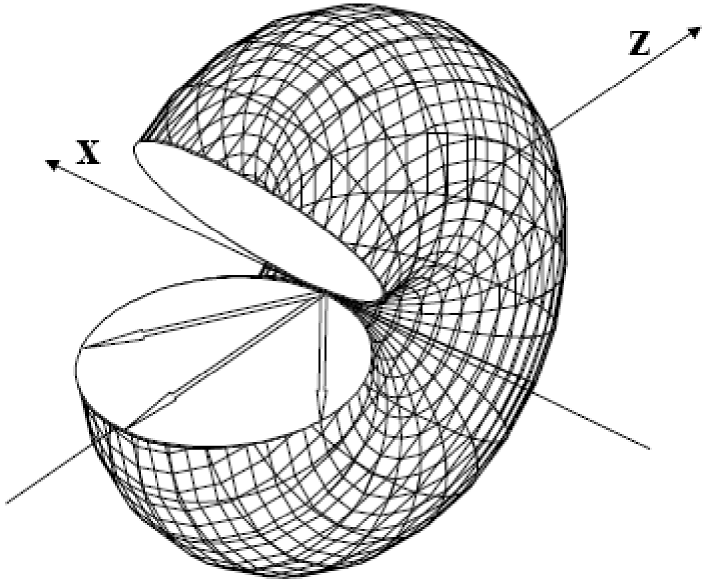
Angular distribution of radiated power: non relativistic motion

Assuming $\overline{\beta} \approx \overline{0}$ **and substituting the acceleration field**

$$\overline{E}_{acc}(\overline{x},t) = \frac{e}{4\pi\varepsilon_0 c}\left[\frac{\overline{n}\times(\overline{n}\times\dot{\overline{\beta}})}{R}\right]_{ret}$$

$$\frac{d^2P}{d\Omega} = \frac{1}{\mu_0 c}\left|R\overline{E}_{acc}\right|^2 = \frac{e^2}{(4\pi)^2\varepsilon_0\, c}\left|\,\overline{n}\times(\overline{n}\times\dot{\overline{\beta}})\right|^2$$

$$\frac{d^2P}{d\Omega} = \frac{e^2}{(4\pi)^2\,\varepsilon_0 c}\left|\dot{\overline{\beta}}\right|^2 \sin^2\theta$$



θ is the angle between the acceleration and the observation direction

Integrating over the angles gives the total radiated power

$$P = \frac{e^2}{6\pi\varepsilon_0 c}\left|\dot{\overline{\beta}}\right|^2$$

Larmor's formula

Angular distribution of radiated power: relativistic motion

Substituting the acceleration field

$$\frac{d^2P}{d\Omega} = \frac{1}{\mu_o c}\left|R\overline{E}_{acc}\right|^2 = \frac{e^2}{(4\pi)^2 \varepsilon_0 c}\frac{\left|\overline{n}\times\left[(\overline{n}-\overline{\beta})\times\dot{\overline{\beta}}\right]\right|^2}{(1-\overline{n}\cdot\overline{\beta})^5}$$

emission is peaked in the direction of the velocity

The pattern depends on the details of velocity and acceleration but it is dominated by the denominator

Total radiated power: computed either by integration over the angles or by relativistic transformation of the 4-acceleration in Larmor's formula

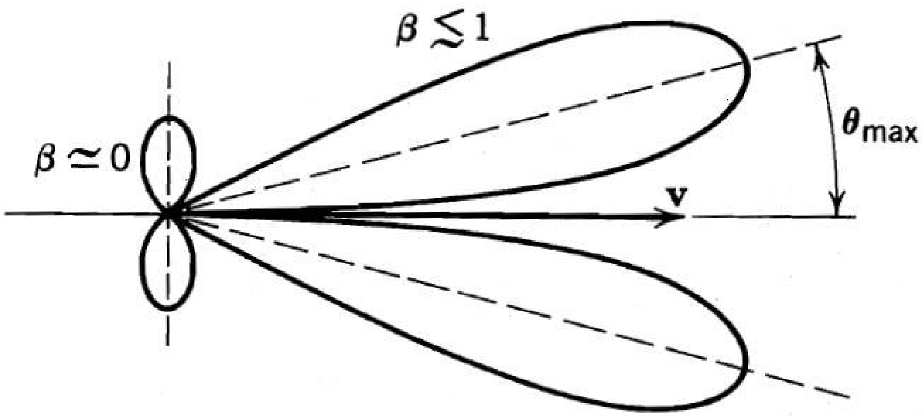
$$P = \frac{e^2}{6\pi\varepsilon_0 c}\gamma^6\left[(\dot{\overline{\beta}})^2 - (\overline{\beta}\times\dot{\overline{\beta}})^2\right]$$

Relativistic generalization of Larmor's formula

velocity || acceleration

Assuming $\bar{\beta} \parallel \dot{\bar{\beta}}$ **and substituting the acceleration field**

$$\frac{dP}{d\Omega} = \frac{e^2}{(4\pi)^2 \varepsilon_0 c} \frac{\left| \bar{n} \times \left[(\bar{n} - \bar{\beta}) \times \dot{\bar{\beta}} \right] \right|^2}{(1 - \bar{n} \cdot \bar{\beta})^5} = \frac{e^2 \left| \dot{\bar{\beta}} \right|^2}{(4\pi)^2 \varepsilon_0 c} \frac{\sin^2 \theta}{(1 - \beta \cos \theta)^5}$$



$$\theta_{\max} = \arccos\left[\frac{1}{3\beta} \left(\sqrt{1 + 15\beta^2} - 1 \right) \right] \to \frac{1}{2\gamma}$$

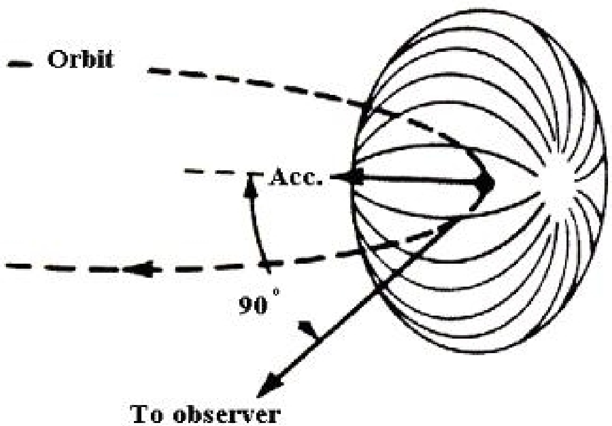
Total radiated power

$$P = \frac{e^2}{6\pi\varepsilon_0 c} \left| \dot{\bar{\beta}} \right|^2 \gamma^6 \qquad P = \frac{e^2}{6\pi\varepsilon_0 m^2 c^3} \left| \frac{d\bar{p}}{dt} \right|^2$$

velocity ⊥ acceleration: synchrotron radiation

Assuming $\bar{\beta} \perp \dot{\bar{\beta}}$ **and substituting the acceleration field**

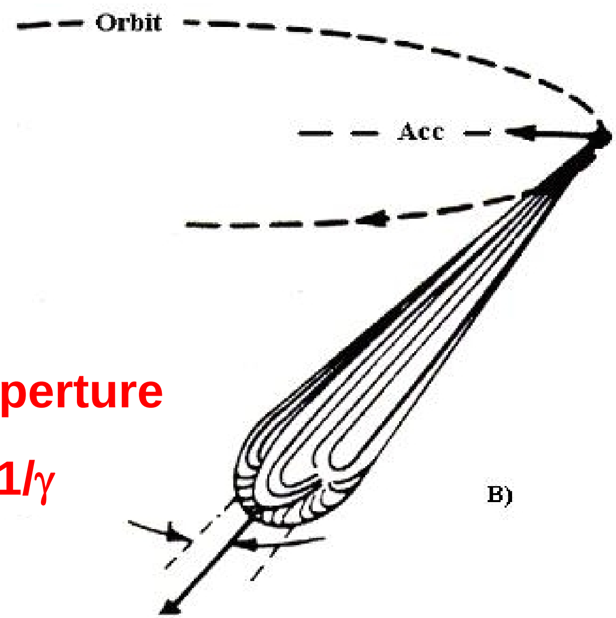
$$\frac{dP}{d\Omega} = \frac{e^2}{(4\pi)^2 \varepsilon_0 c^2} \frac{\left| \bar{n} \times \left[(\bar{n} - \bar{\beta}) \times \dot{\bar{\beta}} \right] \right|^2}{(1 - \bar{n} \cdot \bar{\beta})^5} = \frac{e^2 \left| \dot{\bar{\beta}} \right|^2}{(4\pi)^2 \varepsilon_0 c} \frac{1}{(1 - \beta \cos\theta)^3} \left[1 - \frac{\sin^2\theta \cos^2\phi}{\gamma^2 (1 - \beta\cos\theta)^2} \right]$$



A)

cone aperture

~ 1/γ



B)

When the electron velocity approaches the speed of light the emission pattern is sharply collimated forward

Total radiated power via synchrotron radiation

Integrating over the whole solid angle we obtain the total instantaneous power radiated by one electron

$$P = \frac{e^2}{6\pi\varepsilon_0 c}\left|\dot{\bar{\beta}}\right|^2 \gamma^4 = \frac{e^2}{6\pi\varepsilon_0 c}\left|\dot{\bar{\beta}}\right|^2 \frac{E}{E_o^4}^4 = \frac{e^2}{6\pi\varepsilon_0 m^2 c^3}\left|\frac{d\bar{p}}{dt}\right|^2 \gamma^2 = \frac{e^2 c}{6\pi\varepsilon_0}\frac{\gamma^4}{\rho^2} = \frac{e^4}{6\pi\varepsilon_0 m^4 c^5} E^2 B^2$$

- **Strong dependence $1/m^4$ on the rest mass**
- **$P(v \perp a) \approx \gamma^2 P(v \parallel a)$**
- **proportional to $1/\rho^2$ (ρ is the bending radius)**
- **proportional to B^2 (B is the magnetic field of the bending dipole)**

The radiation power emitted by an electron beam in a storage ring is very high.

The surface of the vacuum chamber hit by synchrotron radiation must be cooled.

Energy loss via synchrotron radiation emission in a storage ring

In the time T_b spent in the bendings the particle loses the energy U_0

$$U_0 = \int P dt = P T_b = P \frac{2\pi\rho}{c} = \frac{e^2}{3\varepsilon_0} \frac{\gamma^4}{\rho}$$

i.e. Energy Loss per turn (per electron)

$$U_0(keV) = \frac{e^2\gamma^4}{3\varepsilon_0\rho} = 88.46 \frac{E(GeV)^4}{\rho(m)}$$

Power radiated by a beam of average current I_b: this power loss has to be compensated by the RF system

$$N_{tot} = \frac{I_b \cdot T_{rev}}{e}$$

$$P(kW) = \frac{e\gamma^4}{3\varepsilon_0\rho} I_b = 88.46 \frac{E(GeV)^4 I(A)}{\rho(m)}$$

Power radiated by a beam of average current I_b in a dipole of length L (energy loss per second)

$$P(kW) = \frac{e\gamma^4}{6\pi\varepsilon_0\rho^2} L I_b = 14.08 \frac{L(m) I(A) E(GeV)^4}{\rho(m)^2}$$

The radiation integral (I)

The energy received by an observer (per unit solid angle at the source) is

$$\frac{d^2W}{d\Omega} = \int_{-\infty}^{\infty} \frac{d^2P}{d\Omega} dt = c\varepsilon_0 \int_{-\infty}^{\infty} | R\overline{E}(t) |^2 \, dt$$

Using the Fourier Transform we move to the frequency space

$$\frac{d^2W}{d\Omega} = 2c\varepsilon_0 \int_{0}^{\infty} | R\overline{E}(\omega) |^2 \, d\omega$$

Angular and frequency distribution of the energy received by an observer

$$\frac{d^3I}{d\Omega d\omega} = 2\varepsilon_0 c R^2 \left| \hat{\overline{E}}(\omega) \right|^2$$

Neglecting the velocity fields and assuming the observer in the <u>far field</u>: n constant, R constant

$$\frac{d^3I}{d\Omega d\omega} = \frac{e^2}{4\pi\varepsilon_0 4\pi^2 c} \left| \int_{-\infty}^{\infty} \frac{\overline{n} \times \left[(\overline{n} - \overline{\beta}) \times \dot{\overline{\beta}} \right]}{(1 - \overline{n} \cdot \overline{\beta})^2} e^{i\omega(t - \overline{n} \cdot \overline{r}(t)/c)} dt \right|^2$$

Radiation Integral

The radiation integral (II)

The radiation integral can be simplified to [see Jackson]

$$\frac{d^3 I}{d\Omega d\omega} = \frac{e^2\omega^2}{4\pi\varepsilon_0 4\pi^2 c}\left|\int_{-\infty}^{\infty} \bar{n}\times(\bar{n}\times\bar{\beta})e^{i\omega(t-\bar{n}\cdot\bar{r}(t)/c)}dt\right|^2$$

How to solve it?

✓ **determine the particle motion** $\bar{r}(t);\ \bar{\beta}(t);\dot{\bar{\beta}}(t)$

✓ **compute the cross products and the phase factor**

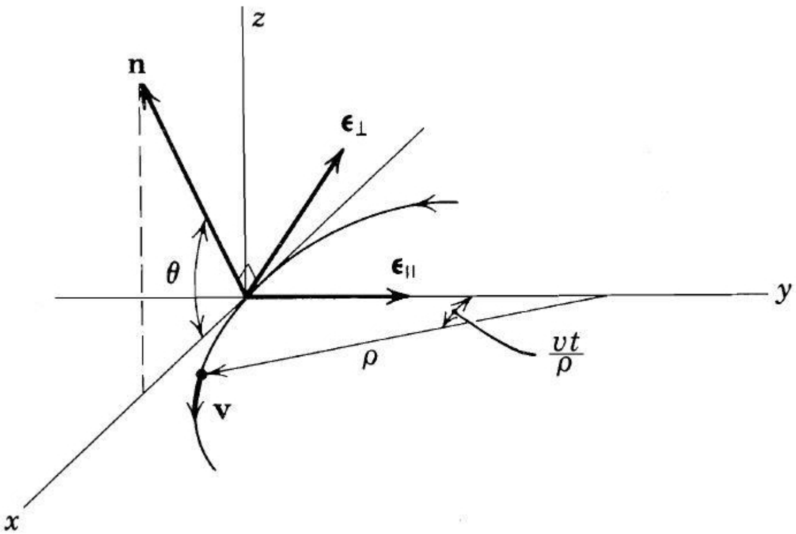
✓ **integrate each component and take the vector square modulus**

Calculations are generally quite lengthy: even for simple cases as for the radiation emitted by an electron in a bending magnet they require Airy integrals or the modified Bessel functions (available in MATLAB)

Radiation integral for synchrotron radiation

Trajectory of the arc of circumference

$$\bar{r}(t) = \left(\rho\left(1 - \cos\frac{\beta c}{\rho} t\right), \quad \sin\frac{\beta c}{\rho} t, \quad 0 \right)$$



In the limit of small angles we compute

$$\bar{n} \times (\bar{n} \times \bar{\beta}) = \beta\left[-\bar{\varepsilon}_{\parallel} \sin\left(\frac{\beta c t}{\rho}\right) + \bar{\varepsilon}_{\perp} \cos\left(\frac{\beta c t}{\rho}\right) \sin\theta \right]$$

$$\omega\left(t - \frac{\bar{n} \cdot \bar{r}(t)}{c} \right) = \omega\left[t - \frac{\rho}{c} \sin\left(\frac{\beta c t}{\rho}\right) \cos\theta \right]$$

Substituting into the radiation integral and introducing $\xi = \frac{\rho\omega}{3c\gamma^3}\left(1 + \gamma^2\theta^2\right)^{3/2}$

$$\frac{d^3 I}{d\Omega\, d\omega} = \frac{e^2}{16\pi^3 \varepsilon_0 c} \left(\frac{2\omega\rho}{3c\gamma^2} \right)^2 \left(1 + \gamma^2\theta^2\right)^2 \left[K_{2/3}^2(\xi) + \frac{\gamma^2\theta^2}{1 + \gamma^2\theta^2} K_{1/3}^2(\xi) \right]$$

Critical frequency and critical angle

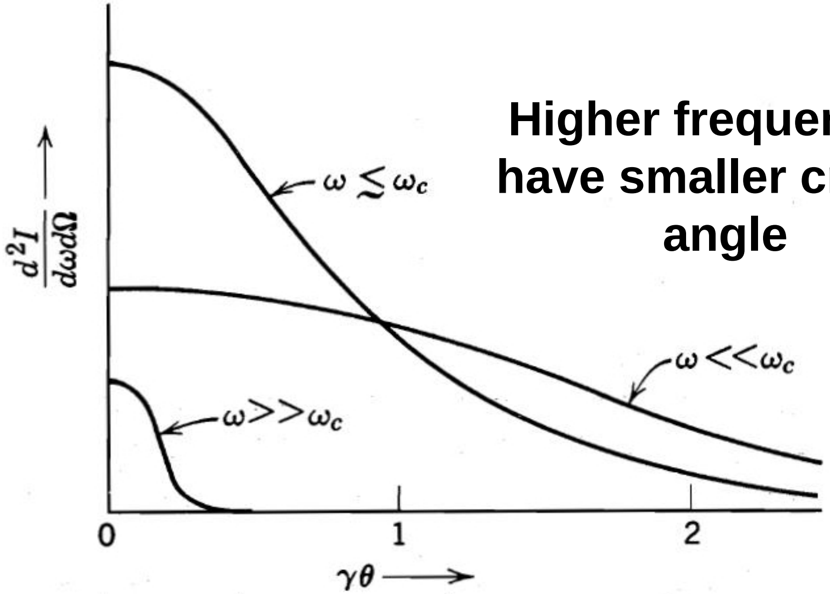
$$\frac{d^3I}{d\Omega\, d\omega} = \frac{e^2}{16\pi^3\varepsilon_0 c}\left(\frac{2\omega\rho}{3c\gamma^2}\right)^2\left(1+\gamma^2\theta^2\right)^2\left[K^2_{2/3}(\xi)+\frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}K^2_{1/3}(\xi)\right]$$

Using the properties of the modified Bessel function we observe that the radiation intensity is negligible for $\xi \gg 1$

$$\xi = \frac{\omega\rho}{3c\gamma^3}\left(1+\gamma^2\theta^2\right)^{3/2} \gg 1$$

Critical frequency $\omega_c = \frac{3}{2}\frac{c}{\rho}\gamma^3$

Critical angle $\theta_c = \frac{1}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3}$



Higher frequencies have smaller critical angle

For frequencies much larger than the critical frequency and angles much larger than the critical angle the synchrotron radiation emission is negligible

Frequency distribution of radiated energy

Integrating on all angles we get the frequency distribution of the energy radiated

$$\frac{dI}{d\omega}=\iint_{4\pi}\frac{d^3I}{d\omega\,d\Omega}d\Omega=\frac{\sqrt{3}e^2}{4\pi\varepsilon_0 c}\gamma\frac{\omega}{\omega_C}\int_{\omega/\omega_C}^{\infty}K_{5/3}(x)dx$$

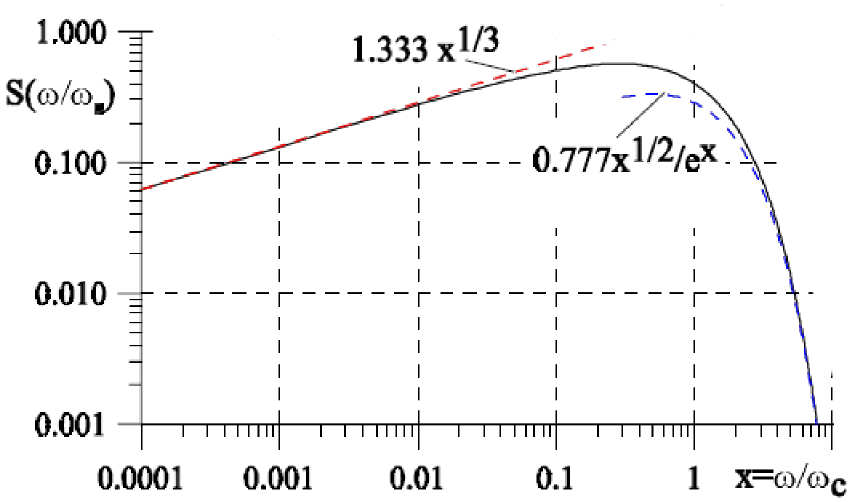
$$\frac{dI}{d\omega}\approx\frac{e^2}{4\pi\varepsilon_0 c}\left(\frac{\omega\rho}{c}\right)^{1/3}\quad \omega \ll \omega_c$$

$$\frac{dI}{d\omega}\approx\sqrt{\frac{3\pi}{2}}\frac{e^2}{4\pi\varepsilon_0 c}\gamma\left(\frac{\omega}{\omega_c}\right)^{1/2}e^{-\omega/\omega_c}\quad \omega \gg \omega_c$$

often expressed in terms of the function S(ξ) with $\xi = \omega/\omega_c$

$$S(\xi)=\frac{9\sqrt{3}}{8\pi}\xi\int_{\xi}^{\infty}K_{5/3}(x)dx \qquad \int_0^{\infty}S(\xi)d\xi=1$$

$$\frac{dI}{d\omega}=\frac{\sqrt{3}e^2\gamma}{4\pi\varepsilon_0 c}\frac{\omega}{\omega_c}\int_{\omega/\omega_c}^{\infty}K_{5/3}(x)dx=\frac{2e^2\gamma}{9\varepsilon_0 c}S(\xi)$$



Frequency distribution of radiated energy

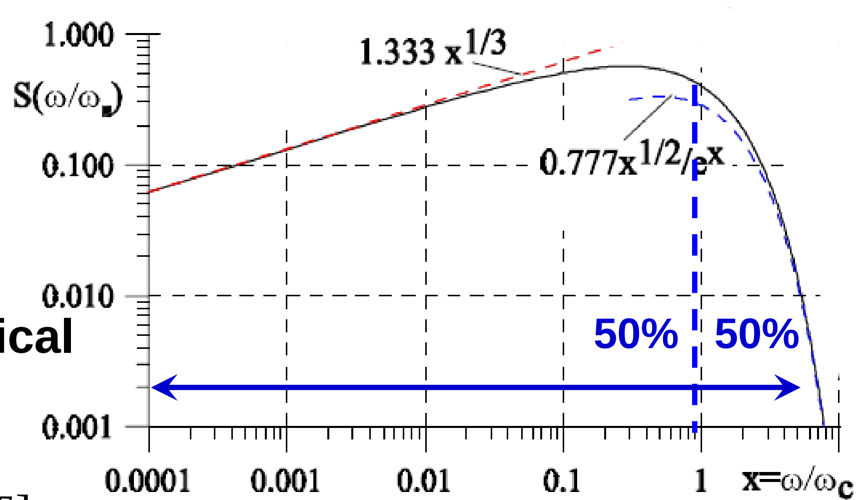
It is possible to verify that the integral over the frequencies agrees with the previous expression for the total power radiated [Hubner]

$$P = \frac{U_0}{T_b} = \frac{1}{T_b}\int_0^{\omega}\frac{dI}{d\omega}d\omega = \frac{1}{T_b}\frac{2e^2\gamma}{9\varepsilon_0 c}\omega_c\int_0^{\omega}\xi\, d\xi\int_{\xi}^{\infty}K_{5/3}(x)dx = \frac{e^2 c}{6\varepsilon_0 c}\frac{\gamma^4}{\rho^2}$$

The frequency integral extended up to the critical frequency contains half of the total energy radiated, the peak occurs approximately at 0.3ω_c

It is also convenient to define the critical photon energy as

$$\varepsilon_c = \hbar\omega_c = \frac{3}{2}\frac{\hbar c}{\rho}\gamma^3$$

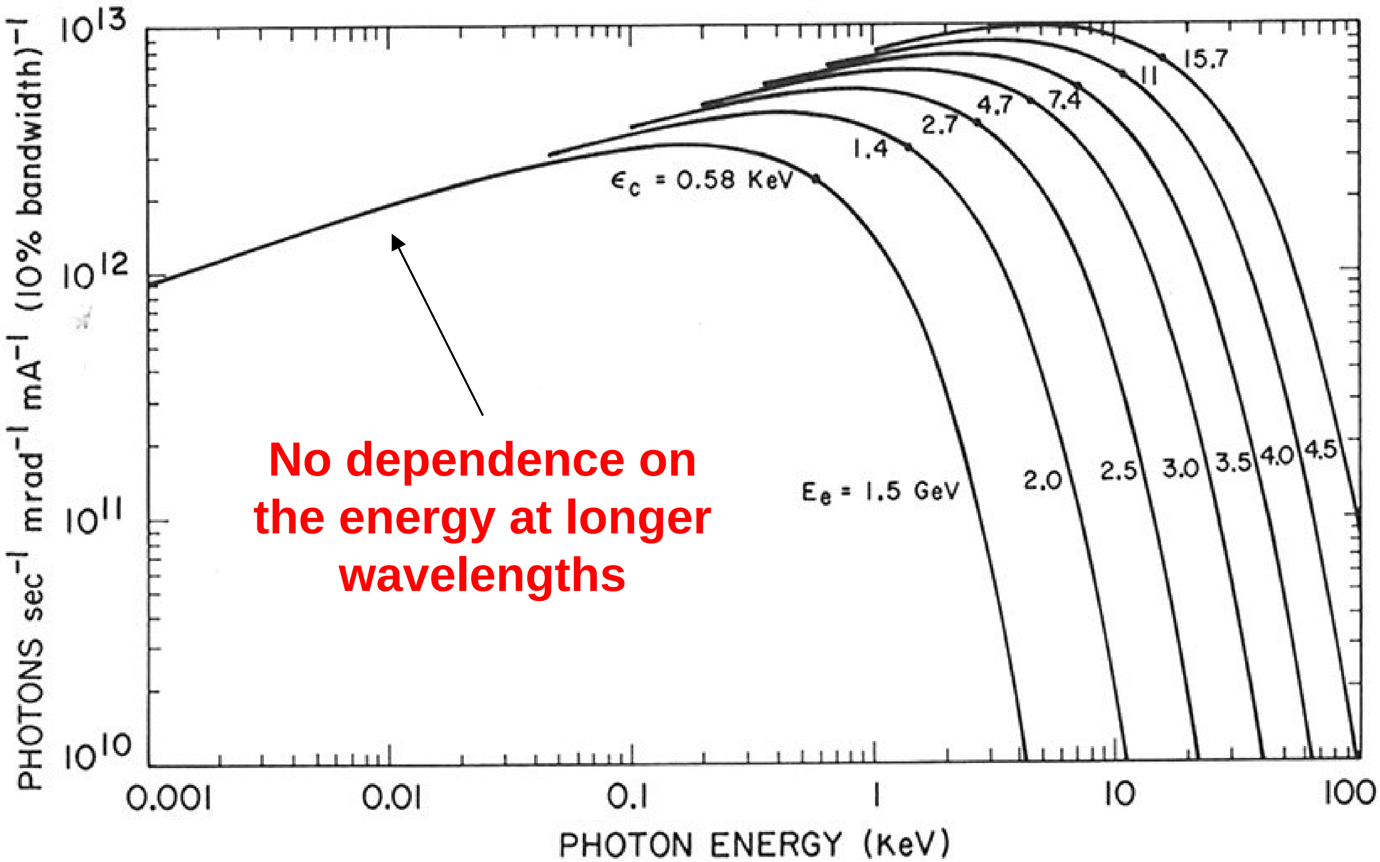


For electrons, the critical energy in practical units reads

$$\varepsilon_c[keV] = 2.218\frac{E[GeV]^3}{\rho[m]} = 0.665\cdot E[GeV]^2\cdot B[T]$$

Synchrotron radiation emission as a function of beam the energy

Dependence of the frequency distribution of the energy radiated via synchrotron emission on the electron beam energy



Polarisation of synchrotron radiation

$$\frac{d^3I}{d\Omega\, d\omega} = \frac{e^2}{16\pi^3\varepsilon_0 c}\left(\frac{2\omega\rho}{3c\gamma^2}\right)^2 \left(1+\gamma^2\theta^2\right)^2 \left[K_{2/3}^2(\xi) + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2} K_{1/3}^2(\xi) \right]$$

Polarisation in the orbit plane

Polarisation orthogonal to the orbit plane

In the orbit plane $\theta = 0$, the polarisation is purely horizontal

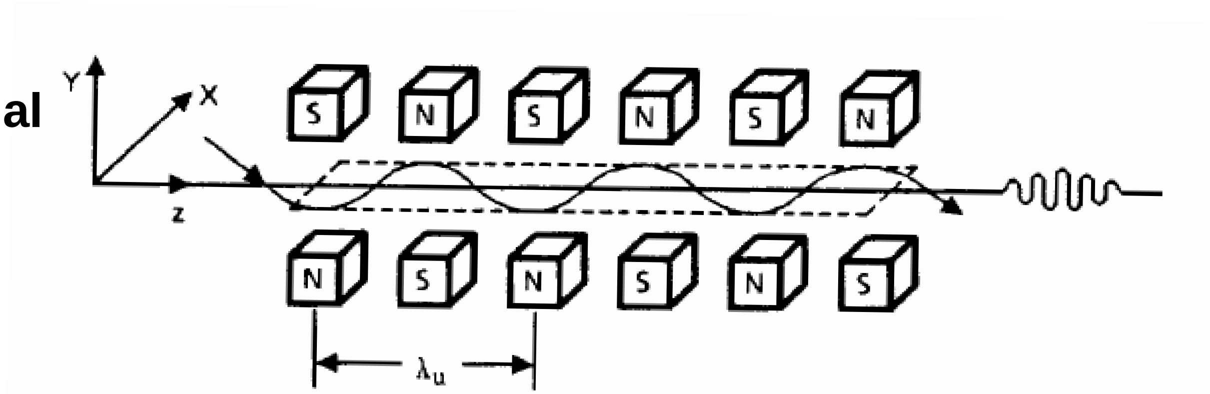
Integrating on all frequencies we get the angular distribution of the energy radiated

$$\frac{d^2I}{d\Omega} = \int_0^\infty \frac{d^3I}{d\omega\, d\Omega} d\omega = \frac{7e^2\gamma^5}{64\pi\varepsilon_0\rho} \frac{1}{(1+\gamma^2\theta^2)^{5/2}} \left[1 + \frac{5}{7}\frac{\gamma^2\theta^2}{1+\gamma^2\theta^2} \right]$$

Integrating on all the angles we get a polarization on the plan of the orbit 7 times larger than on the plan perpendicular to the orbit

Synchrotron radiation from Undulators and wigglers

Periodic array of magnetic poles providing a sinusoidal magnetic field on axis:



$$B = (0, \quad B_0 \sin(k_u z), \quad 0,)$$

Solution of equation of motions:

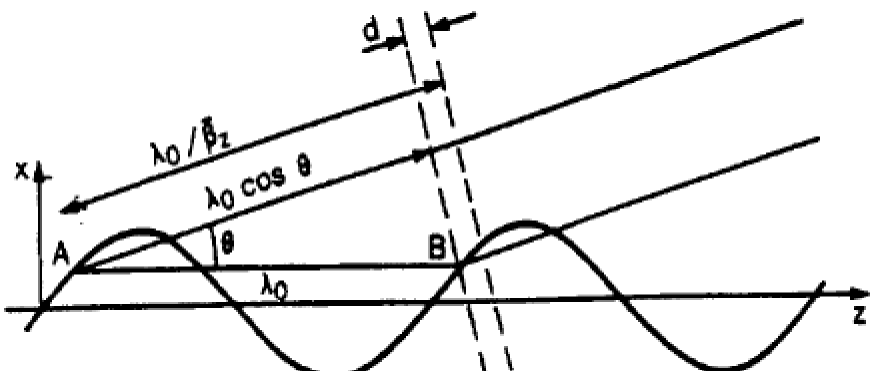
$$K = \frac{eB_0 \lambda_u}{2\pi\, mc}$$

Undulator parameter

$$\bar{r}(t) = -\frac{\lambda_u K}{2\pi\gamma} \sin \omega_u t \cdot \hat{x} + \left(\bar{\beta}_z ct + \frac{\lambda_u K^2}{16\pi\gamma^2} \cos(2\omega_u t) \right) \cdot \hat{z}$$

$$\bar{\beta}_z = 1 - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

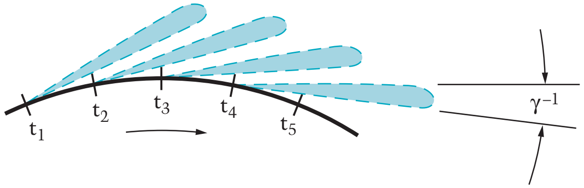
Constructive interference of radiation emitted at different poles



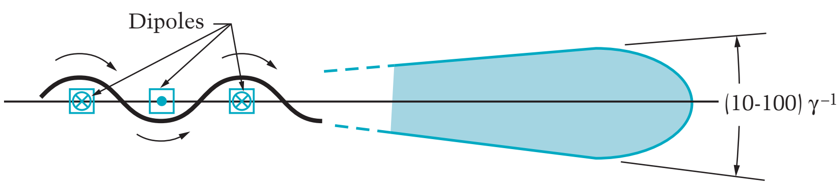
$$d = \frac{\lambda_u}{\bar{\beta}} - \lambda_u \cos\theta = n\lambda$$

$$\lambda_n = \frac{\lambda_u}{2\gamma^2 n}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

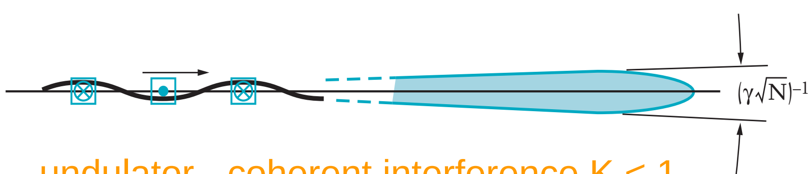
Synchrotron radiation from undulators and wigglers



bending magnet - a “sweeping searchlight”



wiggler - incoherent superposition K > 1



undulator - coherent interference K < 1

Continuous spectrum characterized by ε_c = critical energy

ε_c(keV) = 0.665 B(T)E^2(GeV)

***eg*: for B = 1.4T E = 3GeV ε_c = 8.4 keV**

(bending magnet fields are usually lower ~ 1 – 1.5T)

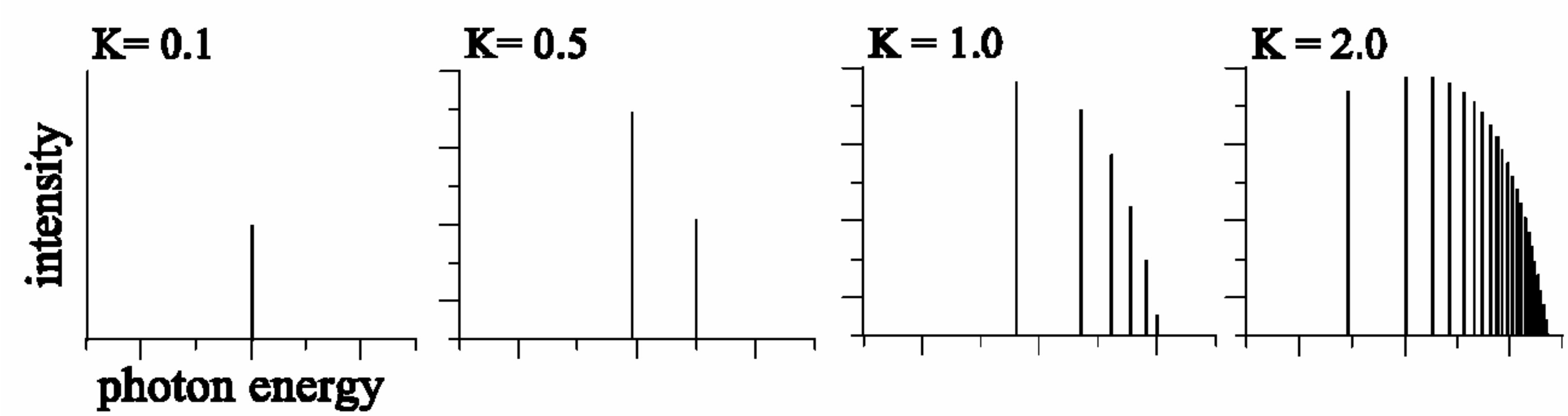
Quasi-monochromatic spectrum with peaks at lower energy than a wiggler

$$\lambda_n = \frac{\lambda_u}{2n\gamma^2}\left(1+\frac{K^2}{2}\right) \approx \frac{\lambda_u}{n\gamma^2}$$

$$\varepsilon_n(eV) = 9.496\frac{nE[GeV]^2}{\lambda_u[m]\left(1+\frac{K^2}{2}\right)}$$

Synchrotron radiation from Undulators and wigglers

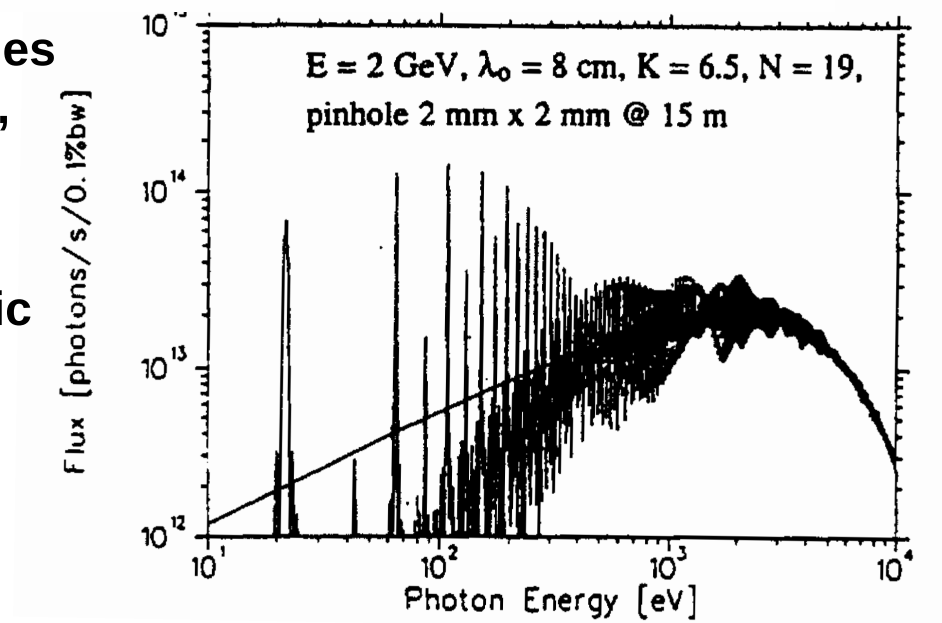
Radiated intensity vs K: as K increases the higher harmonic becomes stronger



For large K the wiggler spectrum becomes similar to the bending magnet spectrum, $2N_u$ times larger.

Fixed B_0, to reach the bending magnet critical wavelength we need the harmonic number n:

K	1	2	10	20
n	1	5	383	3015



Summary

Accelerated charged particles emit electromagnetic radiation

Radiation is stronger for circular orbit

Synchrotron radiation is stronger for light particles

Undulators and wigglers enhance the synchrotron radiation emission

Synchrotron radiation has unique characteristics

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