Lecture 4 - Magnets

ACCELERATOR PHYSICS

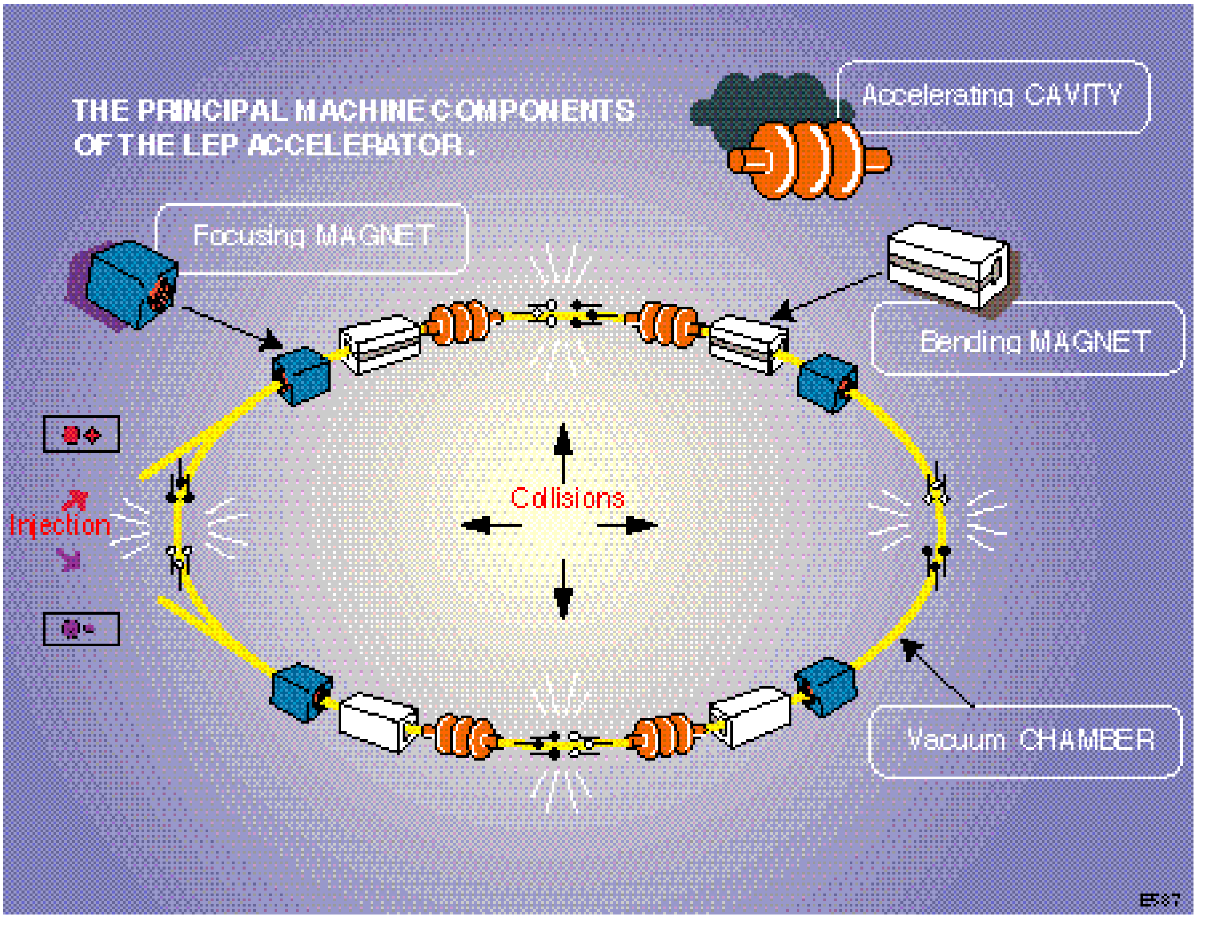
Melbourne

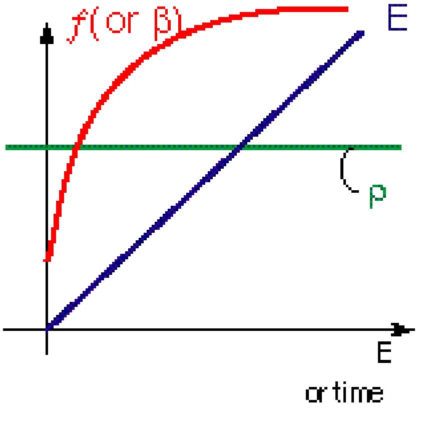
E. J. N. Wilson

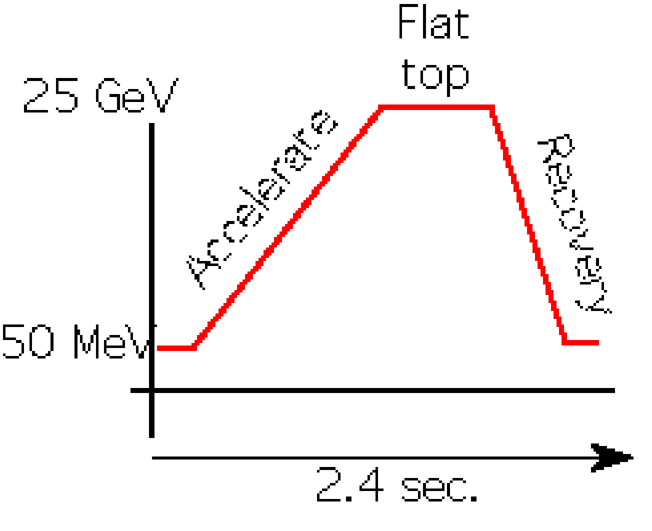
Lecture 4 – Magnets - Contents

- Magnet types
- Multipole field expansion
- Taylor series expansion
- Dipole bending magnet
- Diamond quadrupole
- Various coil and yoke designs
- Power consumption of a magnet
- Magnet cost v. field
- Coil design geometry
- Field quality
- Shims extend the good field
- Flux density in the yoke
- Magnet ends
- Superconducting magnets
- Magnetic rigidity
- Bending Magnet
- Fields and force in a quadrupole

Components of a synchrotron

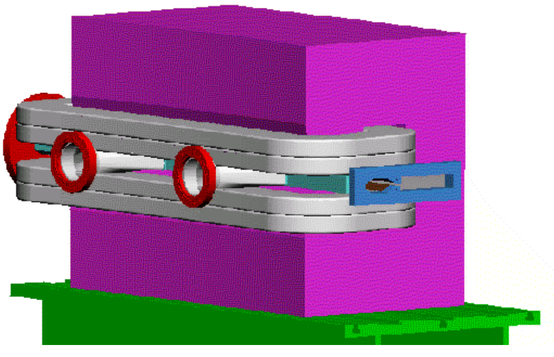


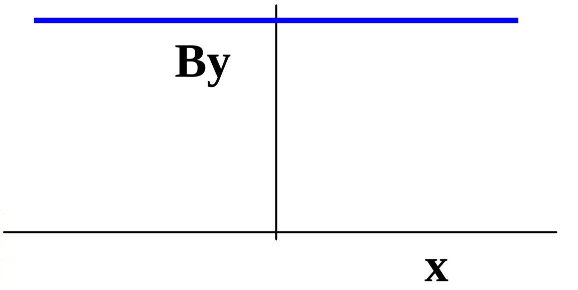




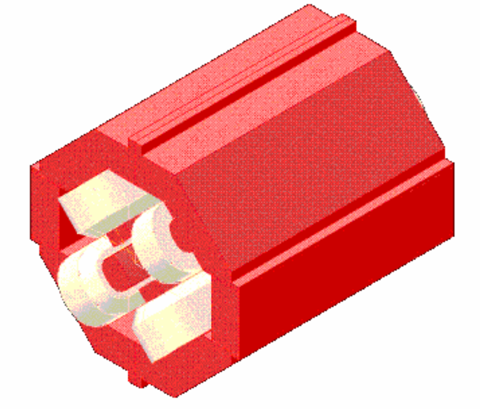
- **RING.GIF, Fig. sans nom 1_PULSE, Annexe1C**

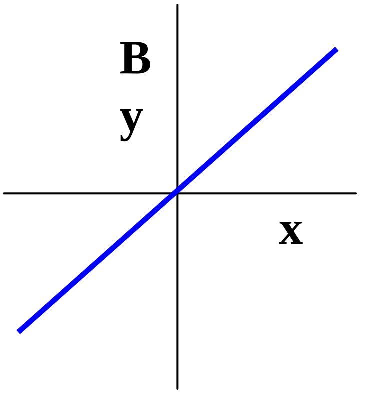
Magnet types



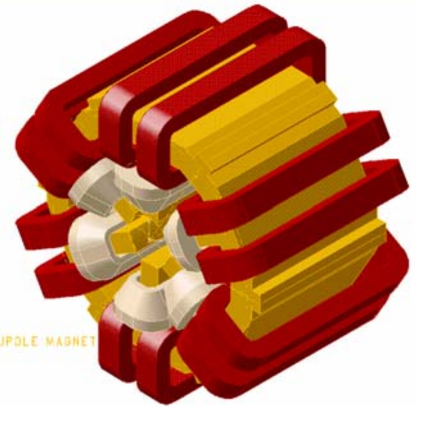


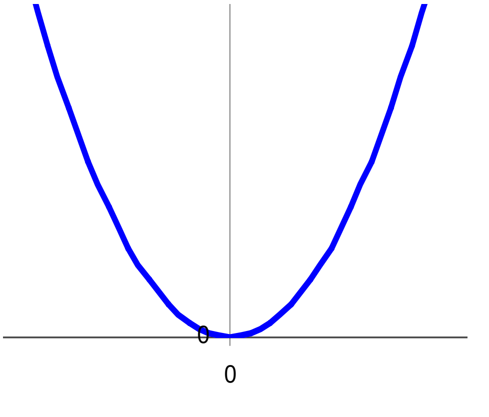
- Dipoles bend the beam





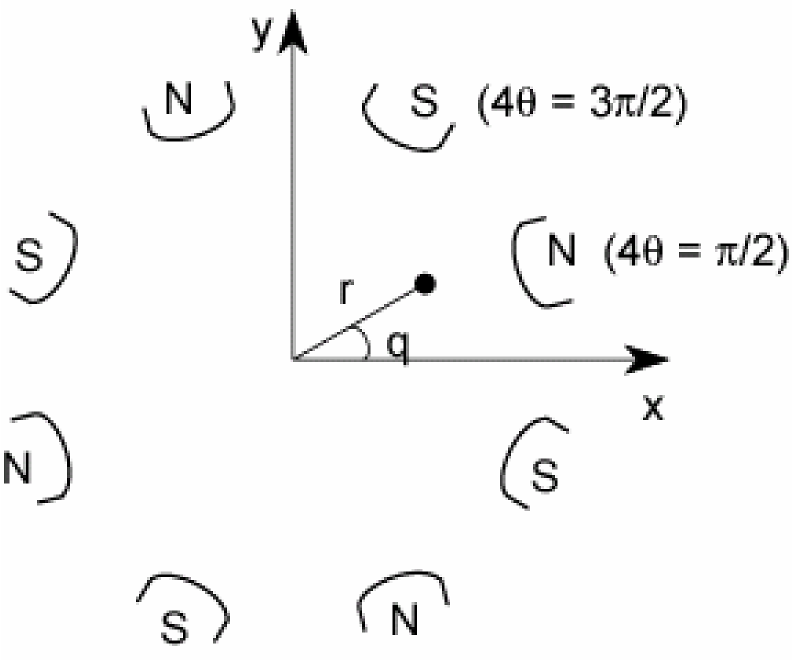
- Quadrupoles focus it





- Sextupoles correct chromaticity

Multipole field expansion (polar)



Scalar potential $\phi(\mathrm{r},\theta)$ **obeys Laplace**

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0 \quad \text{or} \quad \frac{1}{\mathrm{r}^2}\frac{\partial^2\phi}{\partial\theta^2}+\frac{1}{\mathrm{r}}\frac{\partial}{\partial \mathrm{r}}\left(\mathrm{r}\frac{\partial\phi}{\partial \mathrm{r}}\right)=0$$

whose solution is $\phi=\sum_{n=1}^{\infty}\phi_n \mathrm{r}^n \sin n\theta$

Example of an octupole whose potential oscillates like sin 4θ around the circle

Taylor series expansion

$$\phi = \sum_{n=1}^{\infty} \phi_n r^n \sin n\theta$$

Field in polar coordinates:

$$B_r = -\frac{\partial \phi}{\partial r}, \quad B_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta}$$

$$B_r = \phi_n n r^{n-1} \sin n\theta, \quad B_\theta = \phi_n n r^{n-1} \cos n\theta$$

To get vertical field

$$B_z = B_r \sin\theta + B_\theta \cos\theta$$

$$= -\phi_n n r^{n-1}[\cos\theta \cos n\theta + \sin\theta \sin n\theta]$$

$$= \phi_n n r^{n-1} \cos(n-1)\theta = \phi_n n x^{n-1} \quad (\text{when } y = 0)$$

Taylor series of multipoles

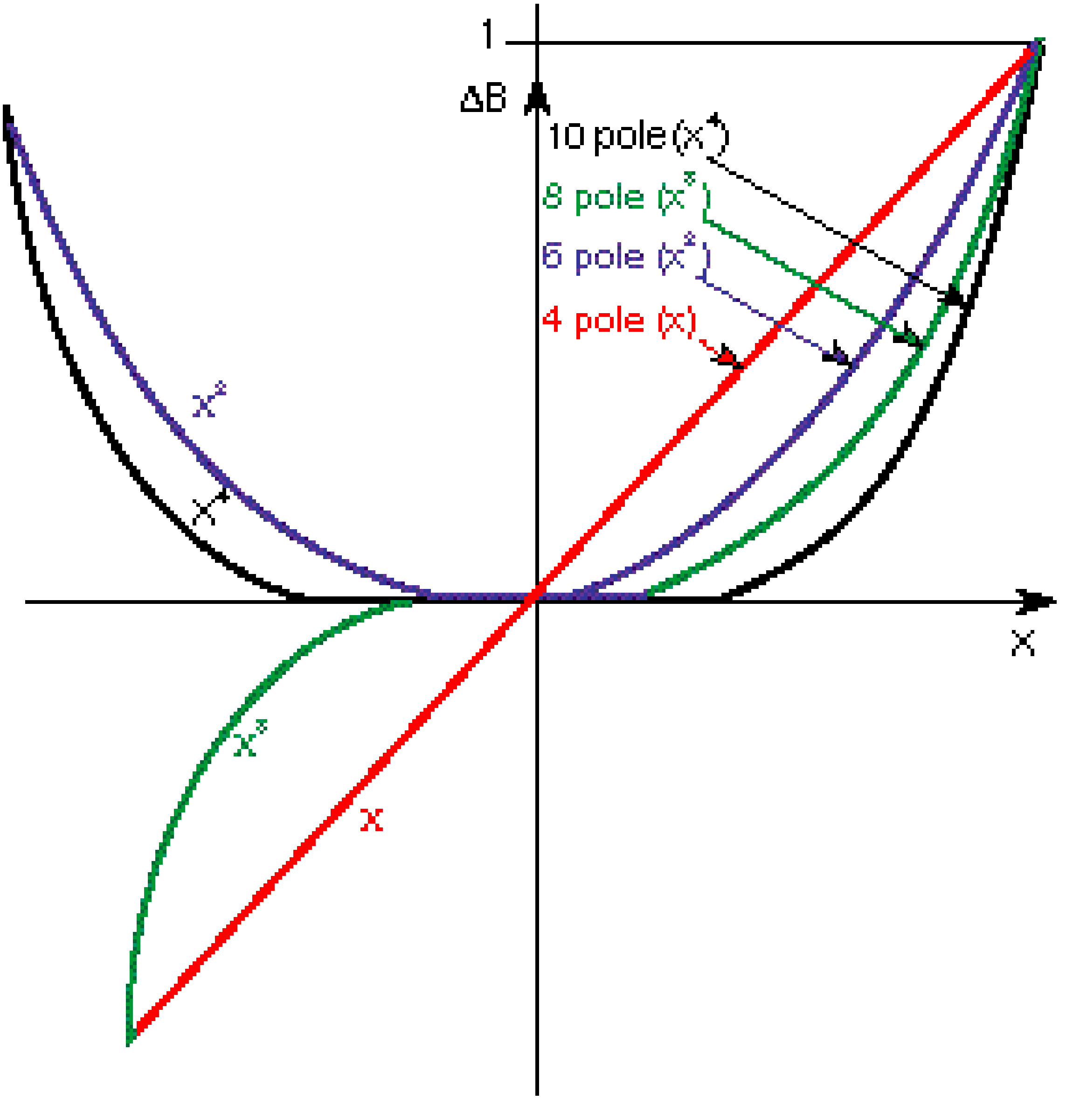
$$B_z = \phi_0 + \phi_2.2x + \phi_3.3x^2 + \phi_4.4x^3 + ...$$

$$= B_0 + \frac{1}{1!}\frac{\partial B_z}{\partial x}x + 2\frac{\partial^2 B_z}{\partial x^2}x^2 + \frac{1}{3!}\frac{\partial^3 B_z}{\partial x^3}x^3 + ...$$

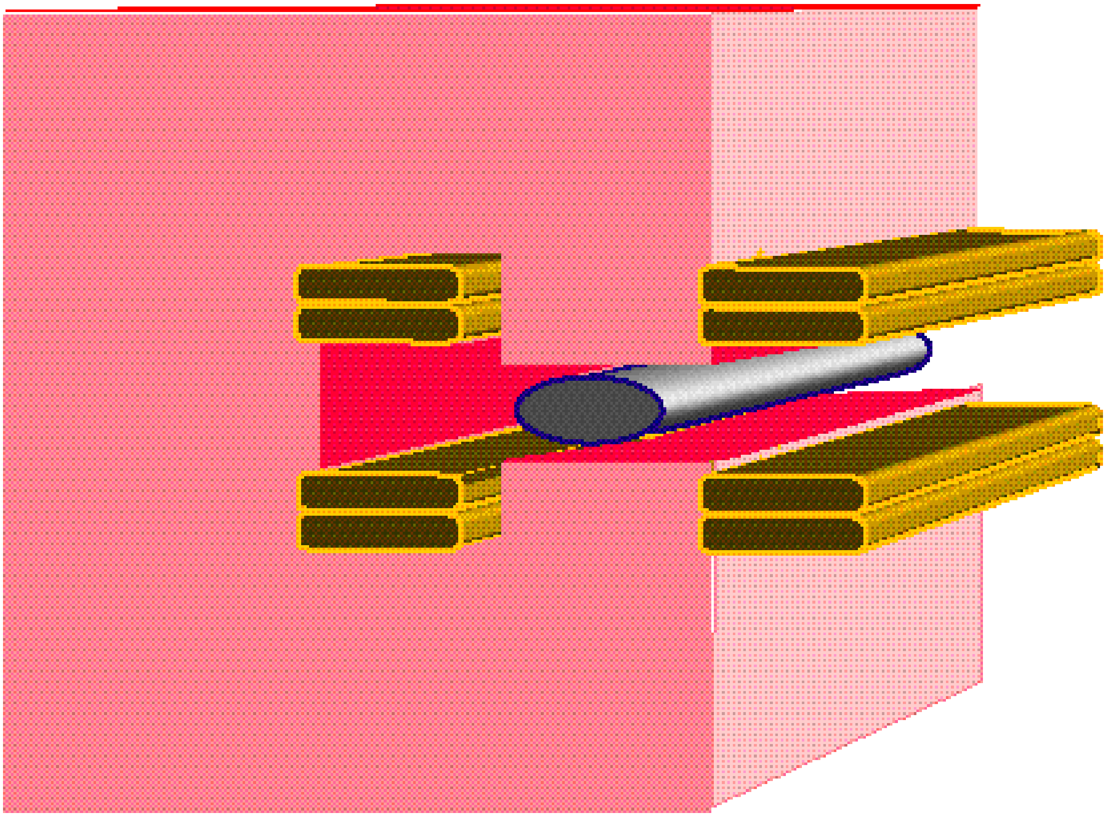
Dip. Quad Sext Octupole

Fig. cas 1.2c

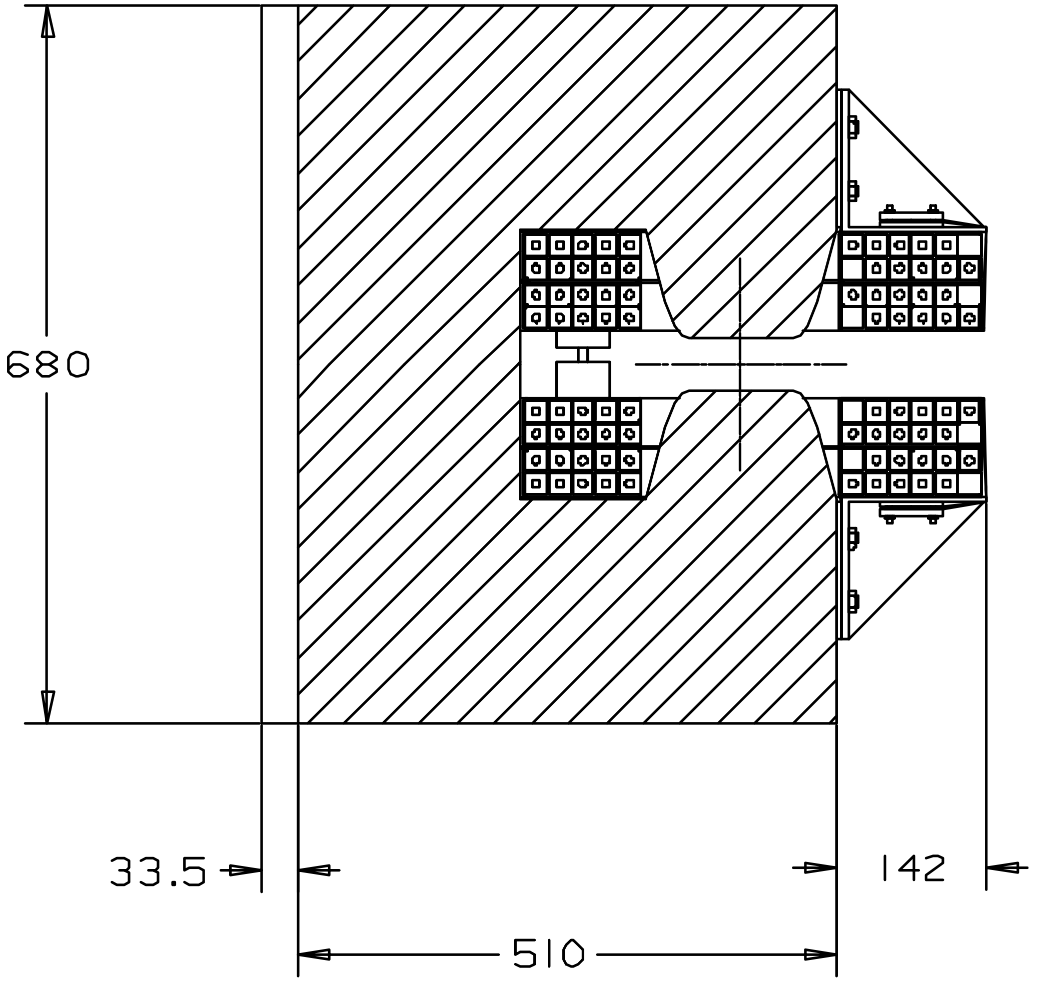
Multipole field shapes



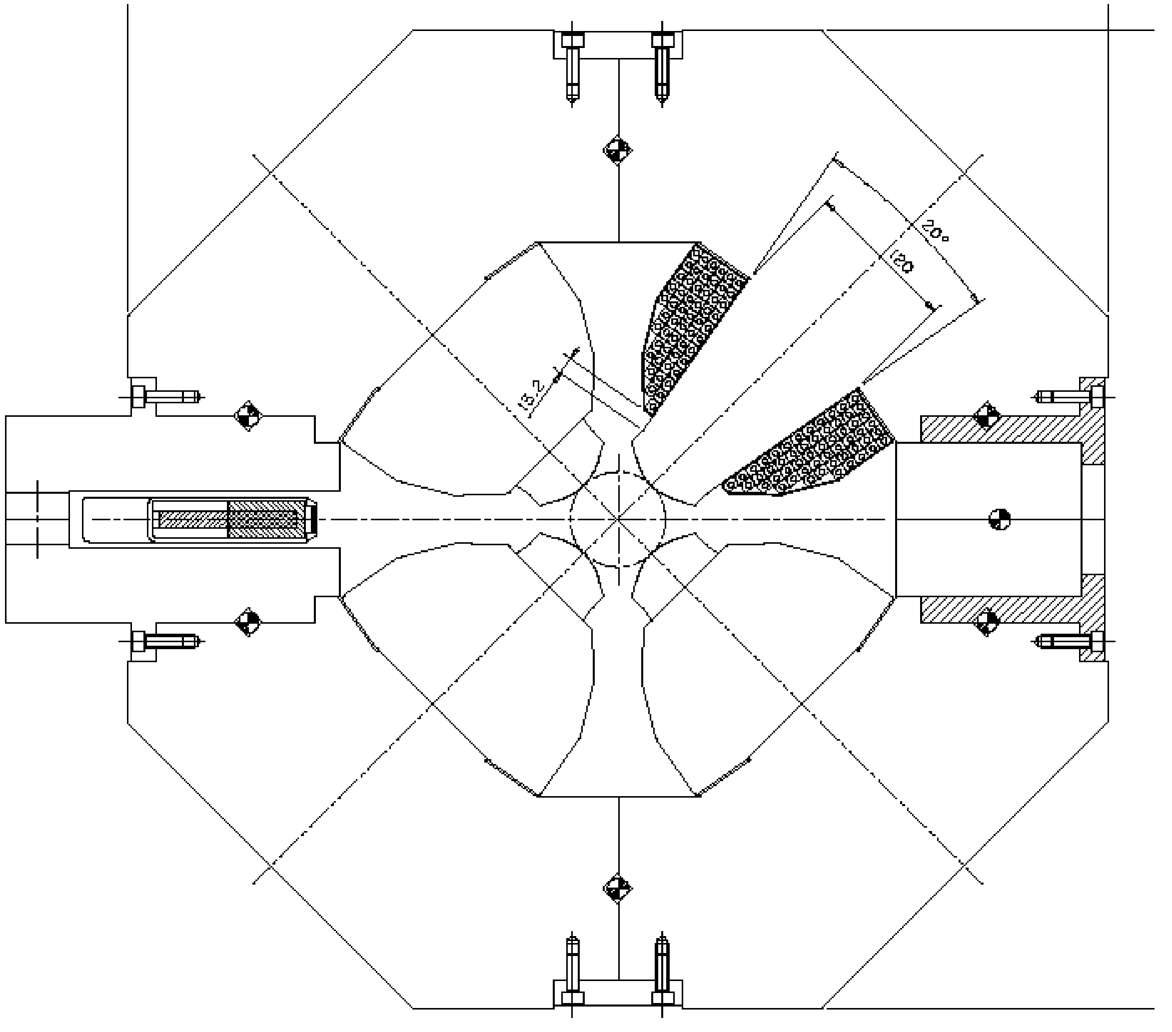
Dipole bending magnet



Diamond dipole



Diamond quadrupole

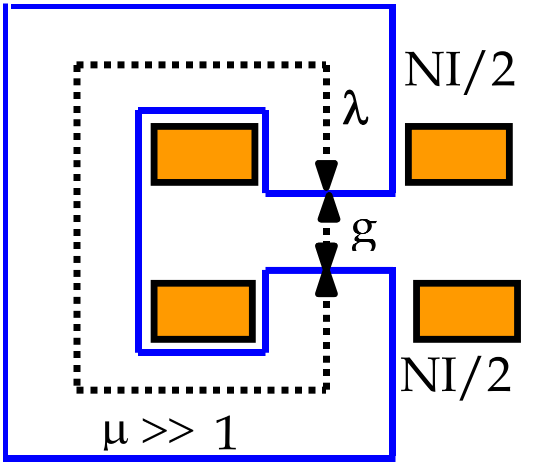


Various coil and yoke designs

- **<u>''C' Core:</u>**

  Easy access

  Less rigid

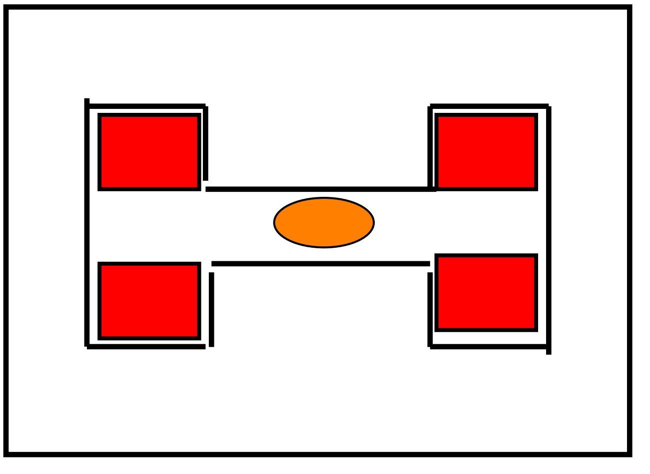


- **‘<u>H core</u>’:**

  Symmetric;

  More rigid;

  Access problems.

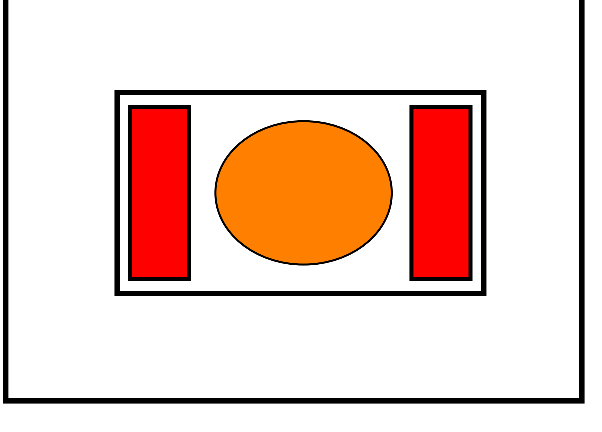


- **''<u>Window Frame</u>'**

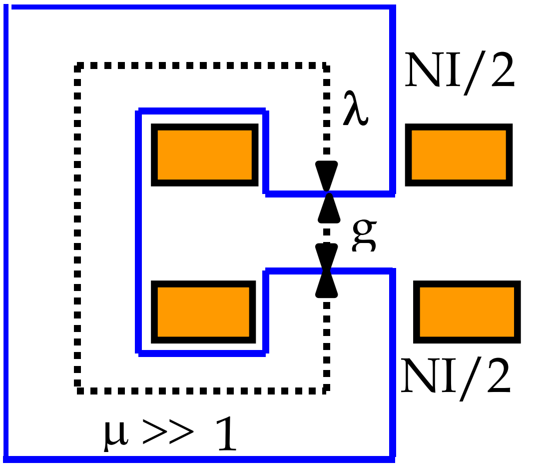
  High quality field;

  Major access problems

  Insulation thickness



Power consumption of a magnet



$B_{air} = \mu_0 NI / (g + \lambda/\mu)$;

g, and λ/μ are the 'reluctance' of the gap and iron. *A* is the coil area, *l* is the length σ is conductivity

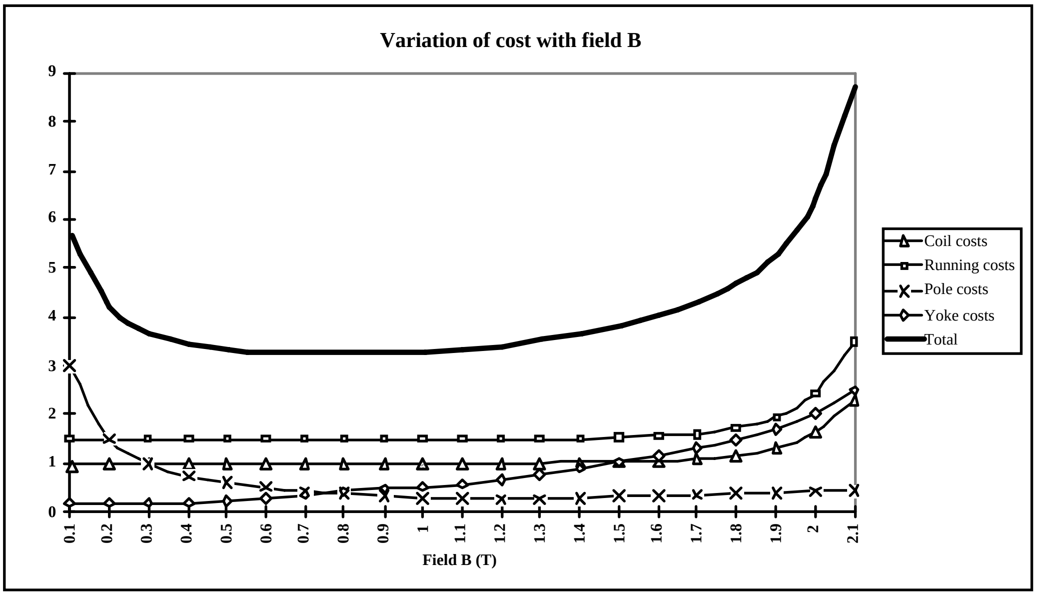
Approximation ignoring iron reluctance ($\lambda/\mu \ll g$):

$$NI = B g / \mu_0$$

$$R = \frac{2\ell N^2}{A\sigma}$$

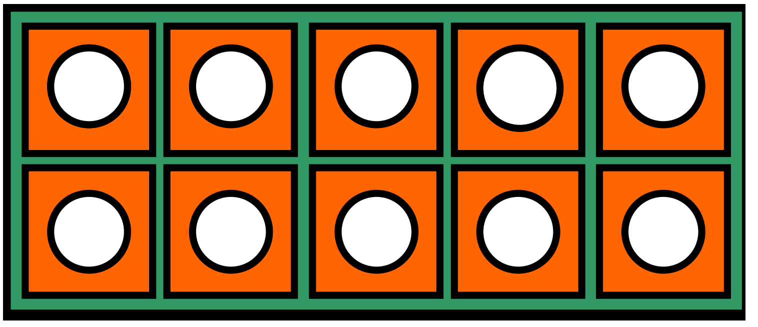
$$\text{Power} = I^2 R = \frac{g^2 B^2 2\ell}{\mu_o^2 A\sigma}$$

Magnet cost v. field



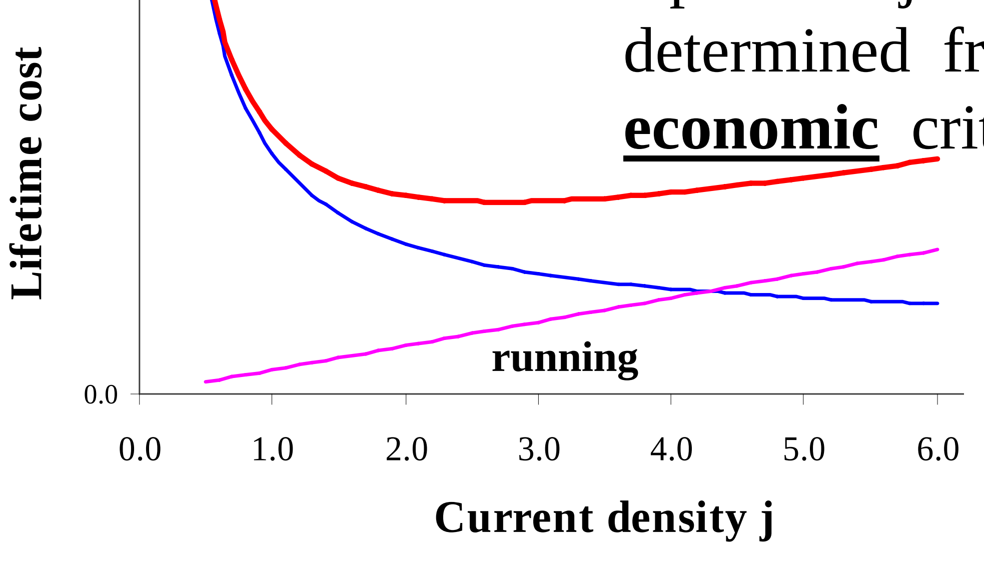
- Pressure from need to save real estate
- Constraint from saturation or critical current, synchrotron radiation

Coil design geometry



- Standard design is rectangular copper (or aluminium) conductor, with cooling water tube. Insulation is glass cloth and epoxy resin.

- Amp-turns (NI) are determined, but total copper area (Acopper) and number of turns (N) are two degrees of freedom and need to be decided.

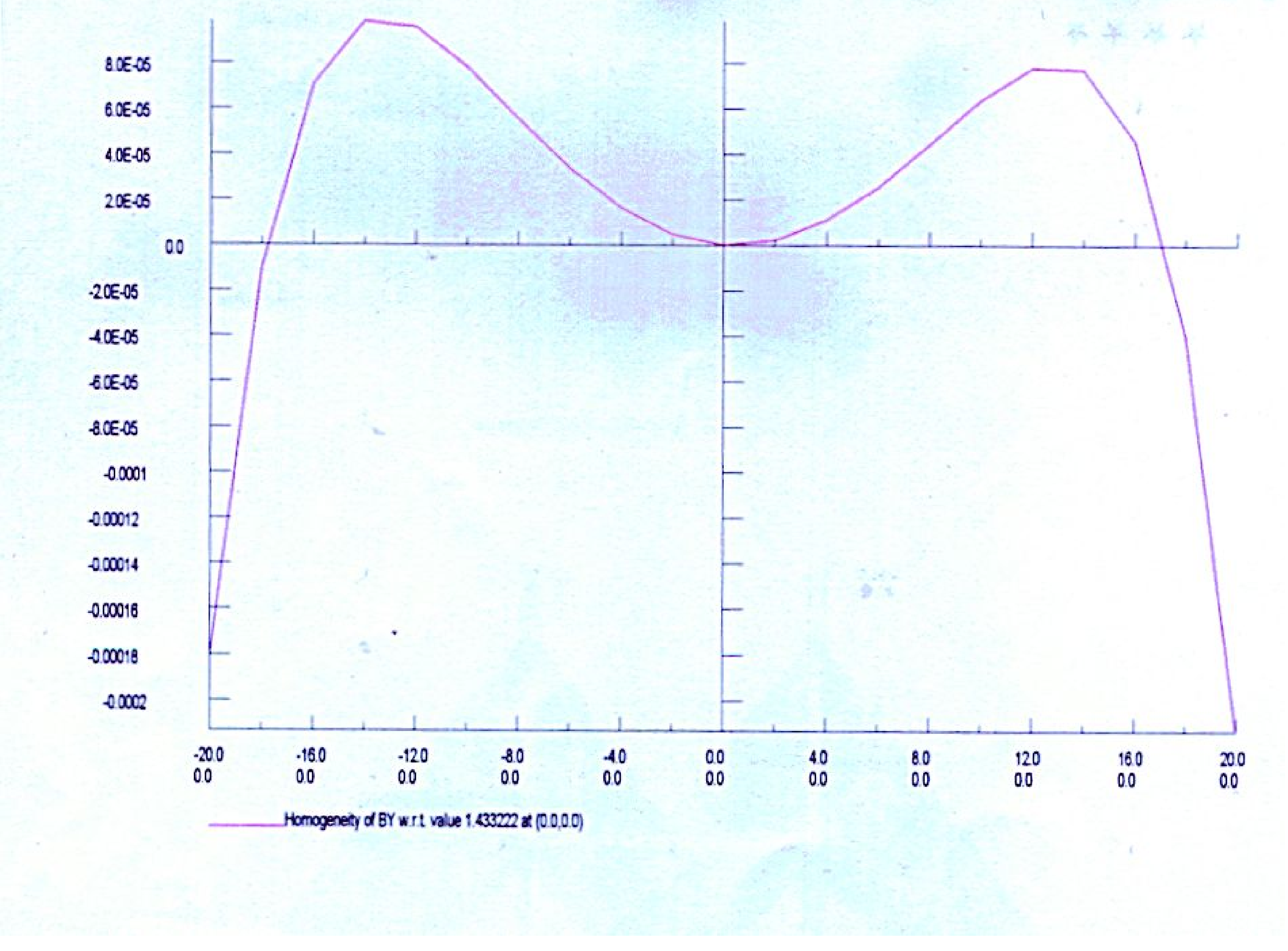


Current density:

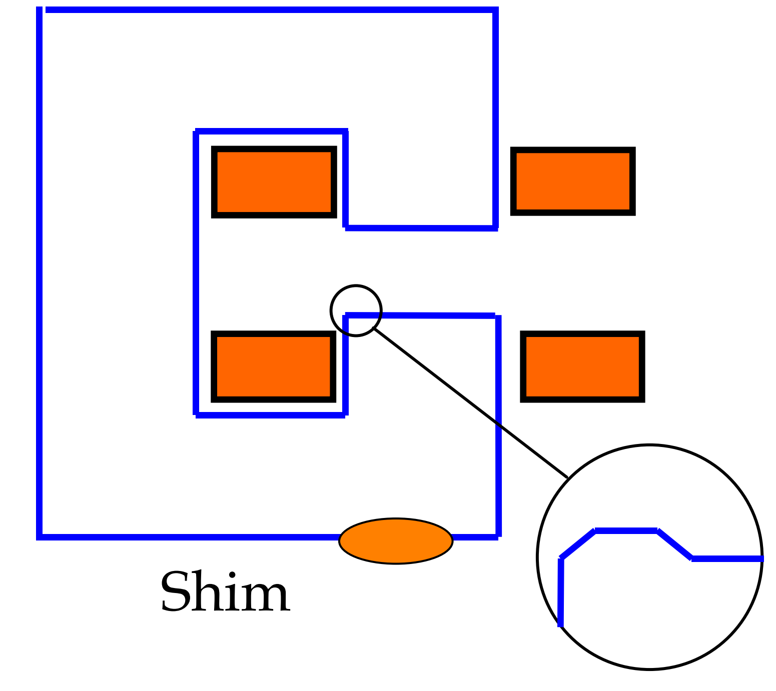
$j = NI/A_{copper}$

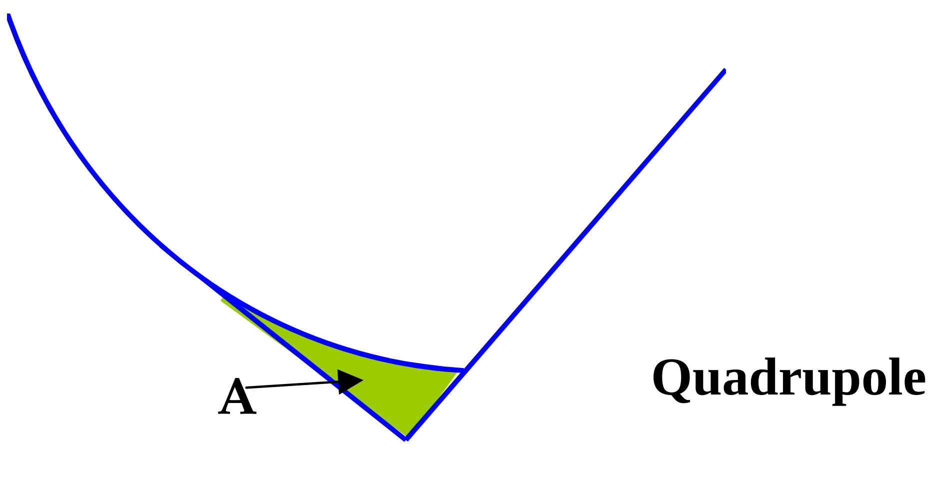
Optimum j determined from **economic** criteria.

Field must be flat to 1 part per 10000



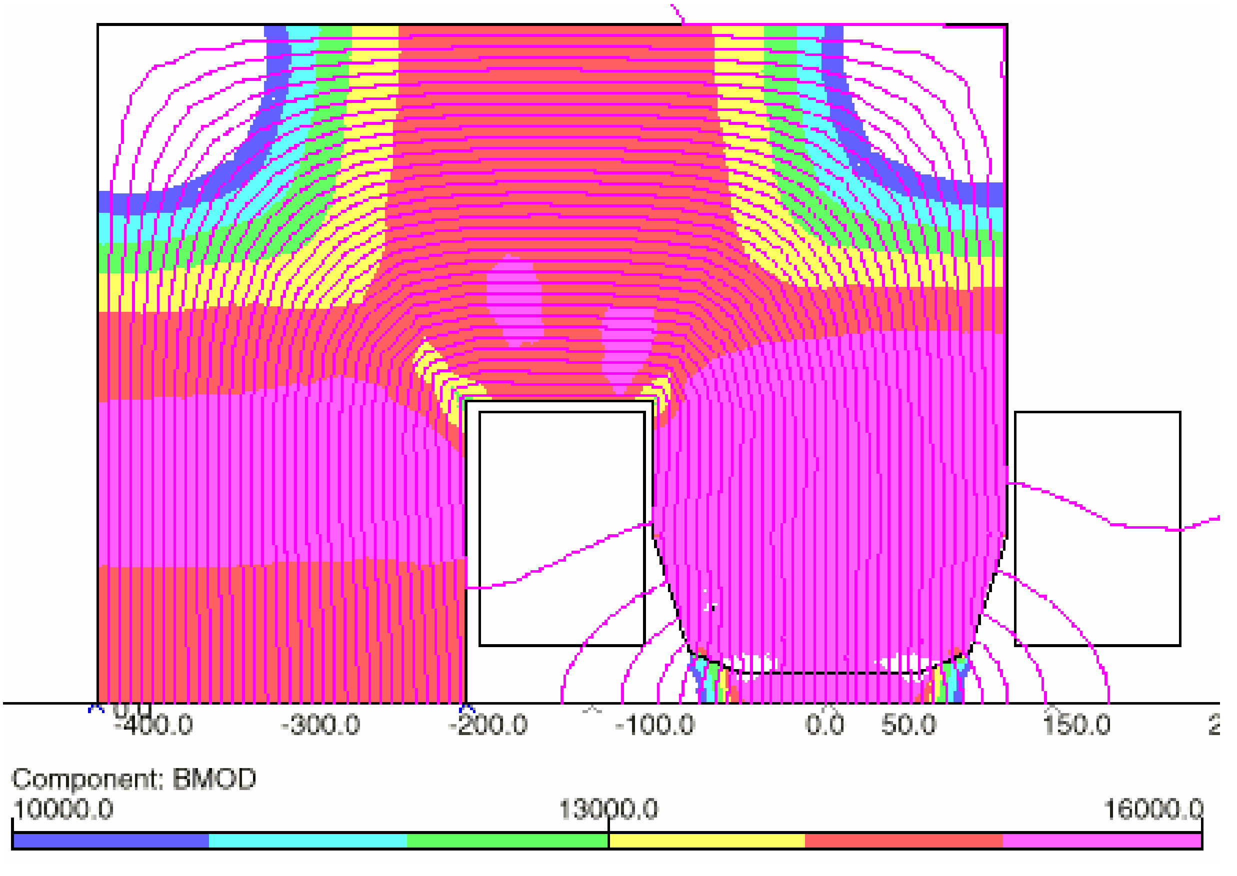
Shims extend the good field



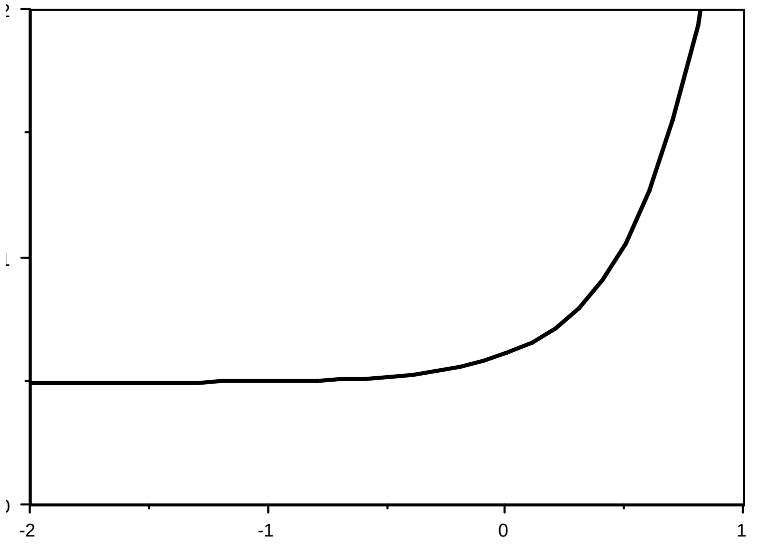


To compensate for the non-infinite pole, shims are added at the pole edges. The area and shape of the shims determine the amplitude of error harmonics which will be present.

Flux density in the yoke



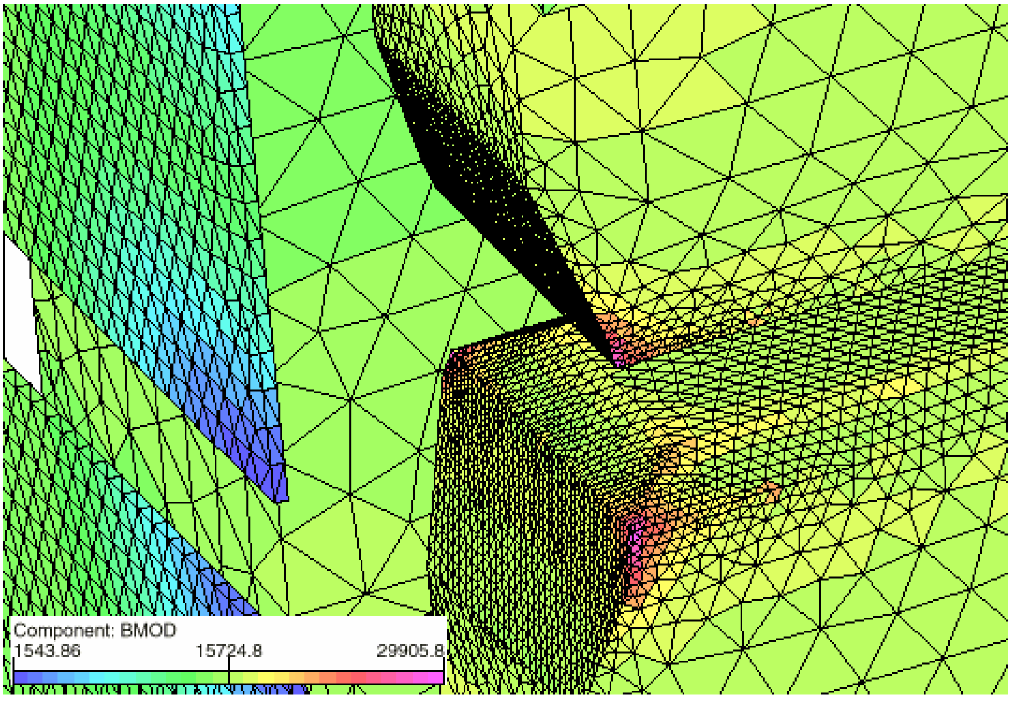
Magnet ends



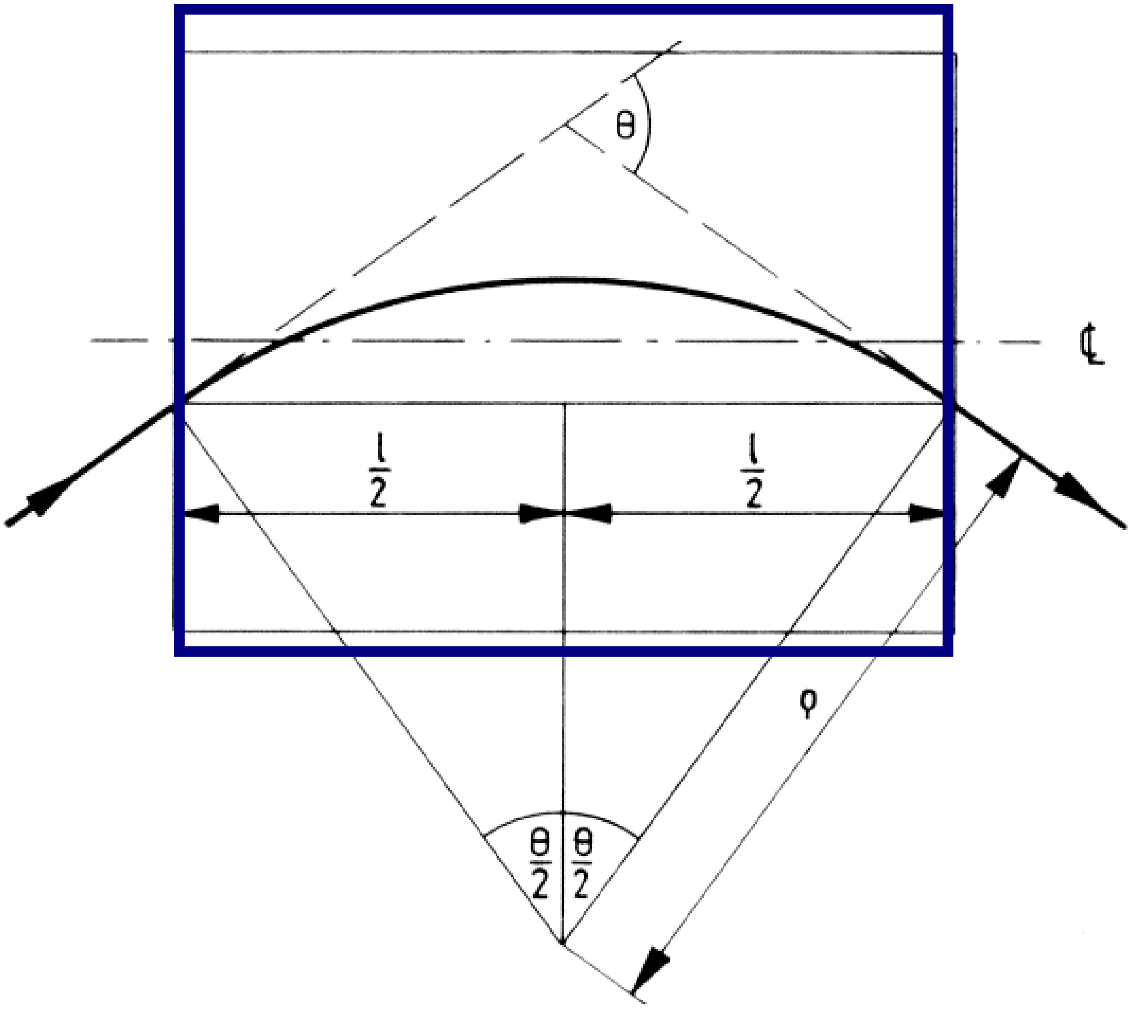
- **The 'Rogowski' roll-off:**

  $y = g/2 + (g/\pi) \exp((\pi x/g)-1);$

- **Three dimensional computer calculation**



Bending Magnet



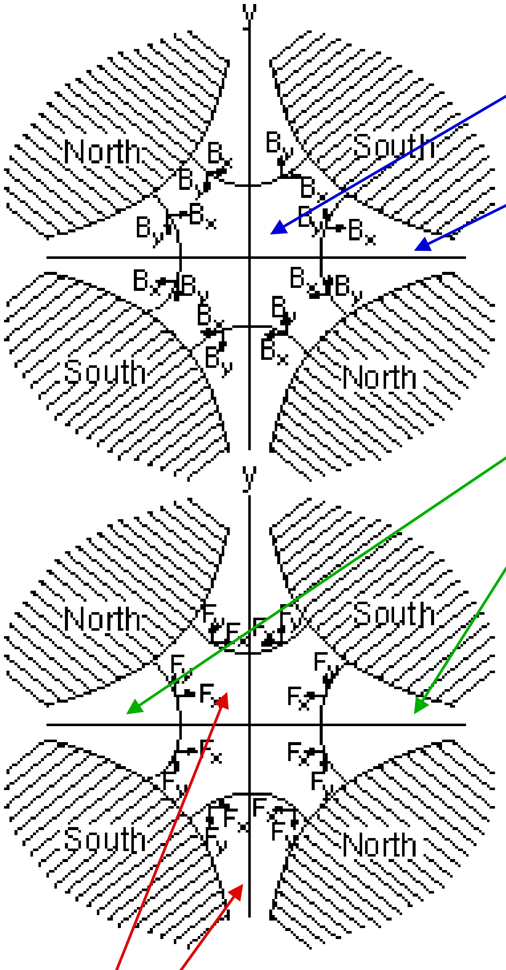
- **Effect of a uniform bending (dipole) field**

$$\sin(\theta/2) = \frac{\ell}{2\rho} = \frac{\ell B}{2(B\rho)}$$

- **If** $\theta << \pi / 2$ **then** $\theta \approx \frac{\ell B}{2(B\rho)}$

- **Sagitta** $\pm \frac{\rho}{2}(1 - \cos(\theta/2)) \approx \frac{\rho\theta^2}{16} = \frac{\ell\theta}{16}$

Fields and force in a quadrupole



No field on the axis

Field strongest here

$$B \propto x$$

(hence is linear)

Force restores

Gradient → $\dfrac{\partial B_y}{\partial x}$

Normalised:

$$k = -\frac{1}{(B\rho)}\cdot\frac{\partial B_y}{\partial x}$$

Defocuses in vertical plane

POWER OF LENS

$$\ell k = -\frac{\ell}{(B\rho)}\cdot\frac{\partial B_y}{\partial x} = \frac{1}{f}$$

Fig. cas 10.8

Summary

- **Magnet types**
- **Multipole field expansion**
- **Taylor series expansion**
- **Dipole bending magnet**
- **Diamond quadrupole**
- **Various coil and yoke designs**
- **Power consumption of a magnet**
- **Magnet cost v. field**
- **Coil design geometry**
- **Field quality**
- **Shims extend the good field**
- **Flux density in the yoke**
- **Magnet ends**
- **Superconducting magnets**
- **Magnetic rigidity**
- **Bending Magnet**
- **Fields and force in a quadrupole**